

**Std. X**

**Subject : Mathematics**

**Part - I and II**



**State Council of Educational Research and Training,  
Maharashtra, Pune.**



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## Std. X - Subject : Mathematics : Part - I and II

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*"Comprehensive Support for Students in Mathematics subject seeking to overcome Past Setbacks."*

**Std. X**

**Subject : Mathematics (Part I and II )**

**OBJECTIVES OF THE BOOKLET**

This booklet is prepared for the help of the students who will be appearing for the Supplementary Examination and thereafter too. It is prepared as such students could not score the minimum score to pass in the written Board examination held in February 2024.

This booklet is designed to boost the confidence of the students. It will definitely help them to score good marks in the forthcoming examination. It will be a great support for the students who lack behind others.

It is prepared in a systematic and easiest way by the expert teachers. The students are aware of the text book as well as the examination pattern (MCQ's, 1 Mark, 2 Marks, 3 Marks and 4 Marks questions). Still, this booklet elaborates every segment in detail. It considers the level of the students.

By studying as suggested in the booklet, we are quite sure that the students will be able to practice a lot with given guidelines. They will score and step into the world of success.

**The main objectives can be summarized as under :**

- 1) To facilitate the essential study material to the students to confidently face the SSC Board Examination.
- 2) To help every low achiever student to achieve 100% success at the SSC Board Examination.
- 3) To motivate the students to score more than their expectation in the Mathematics Subject which they find as most difficult.
- 4) To include tools and exercises that allow students to evaluate their own progress and understand their improvement areas.
- 5) To help the teachers to reach out to students who struggle to pass in the Mathematics subject at the SSC Board Exam with the help of this material.
- 6) Each chapter in the booklet contains important concepts in short.
- 7) Based on these concepts simple solved examples are given.
- 8) Practice questions with hints and answers are given.
- 9) Two practice question papers will definitely help students.



## INTRODUCTION

Dear Students,

It does not matter if you did not score well in the regular examination held in February 2024. Remember, "every setback is a setup for a comeback." Your previous attempt must have taught you something valuable. We believe in your potential to overcome this hurdle and excel in your upcoming exams.

After a comprehensive analysis of the results, SCERT, Maharashtra, Pune has taken an initiative for the upliftment of students who could not achieve the minimum passing score. It was found that some fundamental concepts were not clear to the students. Hence, a significant effort was made to prepare this booklet.

This booklet is designed specifically for those who did not achieve the desired results in their previous Mathematics exam. We understand that facing a setback can be challenging, but it also presents an invaluable opportunity for growth and learning. Our goal with this booklet is to provide you with comprehensive resources and targeted exercises to help you strengthen your understanding of key mathematical concepts. We have carefully curated the content to address common areas of difficulty and to reinforce fundamental principles essential for success in Mathematics.

This booklet will help you to prepare for the supplementary examination. Through a combination of clear explanations, step-by-step problem-solving strategies, and ample practice questions, we aim to build your confidence and competence in the subject. Remember, perseverance and a positive mindset are crucial as you work through this material.

Use this booklet diligently, seek help when needed, and stay committed to your studies. With dedication and effort, you can turn this experience into a stepping stone toward academic success. This resource will also prove to be extremely useful for teachers as they assist students in preparing for the supplementary examination. It will boost your confidence to appear for the exam once again. New students in the coming years can also benefit from this booklet. Best wishes for your journey ahead.

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## Part I

### 1. Linear Equations in Two Variables

- **Let's recall :**

- 1) An equation which contains two variables and the degree of each term containing variable is one, is called a linear equation in two variables.
- 2) General form of a linear equation in two variables :  $ax + by + c = 0$  where a, b, c are real numbers and a, b are not equal to zero at the same time.

- **Practice and Exercise :**

- 1) Which of the following is not a linear equation in two variables.  
(A)  $x + y = 4$       (B)  $x - y = 4$       (C)  $x^2 + y = 4$       (D)  $x = 4y$

**Sol :** (C)  $x^2 + y = 4$ . (As degree of first term is 2.)

- 2) Write the general form of equation  $3x = 4y - 12$ .

**Sol :**  $3x - 4y + 12 = 0$  (Using standard form  $ax + by + c = 0$ )

- 3) Write any two solutions of the equation  $x + y = 7$ .

**Sol :** (0, 7), (7, 0) etc. (Open que.)

- **Simultaneous linear equations :** Two linear equations in the same two variables at the same time, are called simultaneous equations.

- **Simultaneous linear equations can be solved by any of the following methods :**

- (1) Substitution Method
- (2) Elimination Method
- (3) Graphical method
- (4) Determinant method (Cramer's Rule)

- **Practice and Exercise :**

- 1) In the equation  $2x + y = 9$  if  $x = 3$  then  $y =$   
(A) 6      (B) 5      (C) 3      (D) -3

**Sol :** (C) 3

- 2) If  $4x + 3y = 11$  and  $3x + 4y = 10$  then find  $x - y$  ?

**Sol :**  $4x + 3y = 11$

$$\begin{array}{r} 4x + 3y = 11 \\ -3x + 4y = 10 \\ \hline x - y = 1 \end{array}$$

(Subtracting equations)

- 3) Complete the following activity to find value of  $x$  if  $y = 2$  in the equation  $2x - y = 4$  ?

Putting  $y = \boxed{\phantom{00}}$  in equation  $2x - y = 4$

$$\therefore 2x - \boxed{\phantom{00}} = 4$$

$$\therefore 2x = 4 + \boxed{\phantom{00}}$$

$$\therefore x = \boxed{\phantom{00}}$$

- 4) Solve the simultaneous equations :  $x + y = 6$  and  $x - y = 4$

**Sol :**

$$\begin{array}{r} x + y = 6 \quad \dots\dots (I) \\ + \quad x - y = 4 \quad \dots\dots (II) \\ \hline 2x = 10 \quad \text{By adding Equations (I) and (II)} \\ \therefore x = 5 \end{array}$$

Putting  $x = 5$  in equation  $x + y = 6$

$$\therefore 5 + y = 6$$

$$\therefore y = 1$$

$\therefore x = 5, y = 1$  is solution.

- 5) Solve the following simultaneous equations.

$$2x - 3y = 9; \quad 2x + y = 13$$

Solution :  $2x - 3y = 9 \quad \dots\dots(I)$  and  $2x + y = 13 \quad \dots\dots(II)$

Multiply equation (II) by 3

$$\begin{array}{r} 6x + 3y = 39 \quad \dots\dots(III) \\ + 2x - 3y = 9 \quad \dots\dots(I) \\ \hline 8x = 48 \quad \text{(adding equations (I) and (III))} \\ \therefore x = 6 \end{array}$$

Putting  $x = 6$  in equation  $2x + y = 13$

$$\therefore 2 \times 6 + y = 13$$

$$\therefore y = 13 - 12 = 1$$

$\therefore x = 6, y = 1$  is solution.

- 6) If  $15x + 17y = 21$  and  $17x + 15y = 11$ , then find the value of  $x + y$ .

- 7) Solve  $x + y = 5$  ;  $x - y = 3$

• **Graphical method :**

- 1) Draw graphs of the equations.
- 2) Note the co-ordinates of the point of intersection of the two graphs.

• **Practice and Exercise :**

- 1) To draw graph of  $4x + 5y = 19$ , Find  $y$  when  $x = 1$ .

(A) 4                      (B) 3                      (C) 2                      (D) -3

**Sol :** (B) 3

- 2) Complete the following table to draw graph of the equation  $x + y = 3$

$x$	3	<input type="text"/>	<input type="text"/>
$y$	<input type="text"/>	5	3
$(x, y)$	(3, 0)	<input type="text"/>	(0, 3)

**Sol :**

$x$	3	<input type="text" value="-2"/>	<input type="text" value="0"/>
$y$	<input type="text" value="0"/>	5	3
$(x, y)$	(3, 0)	<input type="text" value="-2, 5"/>	(0, 3)

- 3) Complete the following table to draw graph of the equation  $x - y = 4$

$x$	<input type="text"/>	-1	0
$y$	0	<input type="text"/>	-4
$(x, y)$	<input type="text"/>	<input type="text"/>	(0, -4)

• **Determinant :**

- 1)  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$  is a determinant of order 2.
- 2) Determinant represents a number  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a \times d) - (b \times c)$ , (value of determinant)
- 3) Determinant method (Cramer's Rule) : To use Cramer's method,  
Write given equations in the form  $ax + by = c$   
Find the values of determinants  $D$ ,  $D_x$  and  $D_y$   
Using,  $x = \frac{D_x}{D}$  and  $y = \frac{D_y}{D}$  find values of  $x$ ,  $y$ .

• **Practice and Exercise :**

- 1) Find the value of  $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$   
(A) -1      (B) -4      (C) 41      (D) 1

**Sol :**

$$\begin{aligned} \text{(D) 1} \quad \text{as } \begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix} &= (5 \times -4) - (-7 \times 3) \\ &= -20 + 21 \\ &= 1 \end{aligned}$$

- 2) For simultaneous equations in variables  $x$  and  $y$ ,  $D_x = 49$ ,  $D_y = -63$ ,  
 $D = 7$  = then what is  $x$

**Sol :** Using,  $x = \frac{D_x}{D} = \frac{49}{7} = 7$

- 3) Fill in the blanks with correct number

$$\begin{aligned} \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} &= (5 \times \square) - (\square \times 3) \\ &= \square - 21 \\ &= \square \end{aligned}$$

**Sol :**

$$\begin{aligned} &= (5 \times \square{9}) - (\square{7} \times 3) \\ &= \square{45} - 21 \\ &= \square{24} \end{aligned}$$

4)  $ax + by = c$  and  $mx + ny = d$  and  $an \neq bm$  then these simultaneous equations have...

- A) Only one common solution.      B) No Solution.  
C) Infinite number of solutions.      D) Only two solutions.

5) Find the values of the determinant  $\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$

6) To solve  $x + y = 3$  ;  $3x - 2y - 4 = 0$  by determinant method find D and  $D_y$

7) Solve the following simultaneous equations using Cramer's rule.

$$3x - 4y = 10 ; 4x + 3y = 5$$

**Sol :**  $3x - 4y = 10 ; 4x + 3y = 5$

$$\begin{aligned} D &= \begin{vmatrix} 3 & -4 \\ 4 & 3 \end{vmatrix} = (3 \times 3) - (-4 \times 4) \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

$$\begin{aligned} D_x &= \begin{vmatrix} 10 & -4 \\ 5 & 3 \end{vmatrix} = (10 \times 3) - (-4 \times 5) \\ &= 30 + 20 \\ &= 50 \end{aligned}$$

$$\begin{aligned} D_y &= \begin{vmatrix} 3 & 10 \\ 4 & 5 \end{vmatrix} = (3 \times 5) - (10 \times 4) \\ &= 15 - 40 \\ &= -25 \end{aligned}$$

By Cramer's Rule

$$x = \frac{D_x}{D} = \frac{50}{25} = 2 \quad \text{and} \quad y = \frac{D_y}{D} = \frac{-25}{25} = -1$$

$\therefore (x, y) = (2, -1)$  is the solution of the given equations.

8) Solve the following simultaneous equations using Cramer's rule.

$$6x - 3y = -10 ; 3x + 5y - 8 = 0$$

• **Equations reducible to a pair of linear equations in two variables :**

We can create new variables making a proper change in the given variables.

Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

• **Practice and Exercise :**

1) Which of the following is linear equation in two variable.

A)  $2x^2 + y = 9$                       B)  $\frac{3}{x} - \frac{4}{y} = 8$

C)  $7xy - 2 = 0$                       D)  $3x + 4y = 0$

**Sol :** (D)  $3x + 4y = 0$

2) Solve the following equations.

$$\frac{2}{x} + \frac{2}{3y} = \frac{1}{6} \quad ; \quad \frac{3}{x} + \frac{2}{y} = 0$$

**Sol :**  $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$  can be written as  $2\left(\frac{1}{x}\right) + \frac{2}{3}\left(\frac{1}{y}\right) = \frac{1}{6}$

and  $\frac{3}{x} + \frac{2}{y} = 0$  can be written as  $3\left(\frac{1}{x}\right) + 2\left(\frac{1}{y}\right) = 0$

Replacing  $\frac{1}{x}$  by 'm' and  $\frac{1}{y}$  by 'n' we get,

$$2m + \frac{2}{3}n = \frac{1}{6} \text{ multiplying by 6 or } 12m + 4n = 1 \text{ .....(I)}$$

$$\text{And } 3m + 2n = 0 \text{ .....(II)}$$

Multiply equation (II) by 2 we get  $6m + 4n = 0$  .....(III)

$$\text{Also } \underline{-12m + 4n = 1 \text{ .....(I)}}$$

$$-6m \qquad = -1 \text{ (subtracting)}$$

$$\therefore m - \frac{-1}{-6} = \frac{1}{6}$$

Using  $m = \frac{1}{6}$  in  $3m + 2n = 0$  .....(II)

$$\therefore 3\left(\frac{1}{6}\right) + 2n = 0$$

$$\therefore 2n = -\frac{1}{2}$$

$$\therefore n = -\frac{1}{4}$$

But  $m = \frac{1}{x}$  and  $n = \frac{1}{y} \therefore \frac{1}{6} = \frac{1}{x}$  and  $-\frac{1}{4} = \frac{1}{y}$

$\therefore x = 6$  and  $y = -4$  or  $(6, -4)$  is the solution.

3) Solve :  $\frac{5}{x-1} + \frac{1}{y-1} = 2$  ;  $\frac{6}{x-1} - \frac{3}{y-2} = 1$

• **Application of simultaneous equations : Using given conditions assume variables and form linear equations.**

• **Practice and Exercise :**

1) The cost of a notebooks is twice the cost of a pen, a linear equation in two variables to represent this statement as -

- A)  $x + y = 2$       B)  $x + 2 = y$       C)  $x - y = 2$       D)  $y = 2x$

**Sol :** (D)  $y = 2x$

2) From the simultaneous equation in two variables 'sum of two numbers is 21'.

**Sol :** Assume numbers be  $x$  and  $y$  then equation is :  $x + y = 21$

3) The perimeter of a rectangle is 40 cm. The length of the rectangle is more than double its breadth by 2. Complete activity to form linear equations ?

Let length of rectangle be  $x$  cm and breadth by  $y$  cm.

From 1<sup>st</sup> condition       $2(x + y) = \square$        $x + y = \square$  ... (I)

From 2<sup>nd</sup> condition       $x = 2y + \square$        $x - \square = 2$  ... (II)



## 2. Quadratic Equations

### Previous Knowledge :

- 1) General form of quadratic equation :**  $ax^2 + bx + c = 0$  (Here  $x$  is a variable,  $a, b, c$  are real numbers and  $a \neq 0$ )
- 2) Roots or solutions of quadratic equation :** The values of variable for which both the sides of equation are equal is called as root or solution of the equation.
- 3) Methods of solving quadratic equations :**
  - 1) Factorization method
  - 2) Completing square method
  - 3) Formula method
- 4) Formula for solving a quadratic equation :**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- 5) Discriminant :**  $\Delta = b^2 - 4ac$
- 6) Nature of roots of a quadratic equation :**
  - 1) If  $\Delta = 0$  then the roots of quadratic equation are real and equal.
  - 2) If  $\Delta > 0$  then the roots of quadratic equation are real and unequal.
  - 3) If  $\Delta < 0$  then the roots of quadratic equation are not real.
- 7) Relation between roots of the quadratic equation and coefficients :** If  $\alpha$  and  $\beta$  are roots of the equation  $ax^2 + bx + c = 0$  then,
  - 1)  $\alpha + \beta = \frac{-b}{a}$
  - 2)  $\alpha\beta = \frac{c}{a}$
- 8) Formula to obtain a quadratic equation having given roots :**  
 $x^2 - [\alpha + \beta]x + \alpha\beta = 0$

### Examples :

#### Q. 1 A) -

- 1) One of the roots of equation  $x^2 + mx - 5 = 0$  is 2; find  $m$ .

(A) - 2      (B)  $-\frac{1}{2}$       (C)  $\frac{1}{2}$       (D) 2

Let  $x = 2$

$$\therefore x^2 + mx - 5 = 0$$

$$\therefore (2)^2 + m \times 2 - 5 = 0$$

$$\therefore 4 + 2m - 5 = 0$$

$$\therefore 2m - 1 = 0 \quad \therefore 2m = 1$$

$$\therefore m = \frac{1}{2} \quad \therefore \text{Ans : Option (C)}$$



2) The roots of  $x^2 + kx + k = 0$  are real and equal, find  $k$ .

- (A) 0      (B) 4      (C) 0 or 4      (D) 2

$$a = 1, b = k, c = k$$

$$\therefore \Delta = b^2 - 4ac = (k)^2 - 4 \times 1 \times k = k^2 - 4k$$

But roots are real and equal

$$\therefore \Delta = 0$$

$$\therefore k^2 - 4k = 0$$

$$\therefore k(k - 4) = 0$$

$$\therefore k = 0 \text{ or } k - 4 = 0$$

$$\therefore k = 0 \text{ or } k = 4 \quad \therefore \text{Ans : Option (C)}$$

3) Which one is the quadratic equation?

(A)  $\frac{5}{x} - 3 = x^2$

(B)  $x(x + 5) = 4$

(C)  $n - 1 = 2n$

(D)  $\frac{1}{x^2}(x + 2) = x$

4) Out of the following equations, find the equation having the sum of its roots 1.

(A)  $x^2 + x + 1 = 0$

(B)  $3x^2 + x + 3 = 0$

(C)  $3x^2 + 3x + 1 = 0$

(D)  $3x^2 - 3x + 1 = 0$

**Q. 1 B)** 1) Decide the equation  $x + \frac{1}{x} = -2$  is quadratic or not?

$$x + \frac{1}{x} = -2$$

Multiply each term by  $x$

$$\therefore x \times x + x \times \frac{1}{x} = -2 \times x$$

$$\therefore x^2 + 1 = -2x$$

$$\therefore x^2 + 2x + 1 = 0$$

Here maximum index of the variable is 2.

$\therefore$  It is a quadratic equation.

2) Check whether  $x = -1$  is solution of the equation  $x^2 + 4x - 5 = 0$  or not?

Take  $x = -1$

$$\begin{aligned}\text{LHS} &= x^2 + 4x - 5 \\ &= (-1)^2 + 4 \times (-1) - 5 \\ &= 1 - 4 - 5 \\ &= -8\end{aligned}$$

$$\text{RHS} = 0$$

Here LHS  $\neq$  RHS

$\therefore x = -1$  is not a solution of the equation.

3) Write the equation  $(l + 2)(l - 5) = 0$  in the form  $ax^2 + bx + c = 0$

4) Write the values of  $a, b, c$  for the equation  $y^2 = 2y - 7$ .

5) Write a quadratic equation from following information.

'In a garden there are 150 trees. The number of trees in each row is 5 more than that in each column.'

**Q. 2 ) -**

1) Complete the following activity to find the value of discriminant of the equation  $3x^2 + 2x - 1 = 0$ .

**Activity :**

Comparing  $3x^2 + 2x - 1 = 0$  with  $ax^2 + bx + c = 0$

$$\therefore a = 3, b = 2, c = \boxed{\phantom{00}}$$

$$\Delta = b^2 - 4ac$$

$$= (2)^2 - 4 \times 3 \times \boxed{\phantom{00}}$$

$$= 4 + \boxed{\phantom{00}}$$

$$\therefore \Delta = \boxed{\phantom{00}}$$

2) Complete the following activity for solving the quadratic equation,  $x^2 + 8x - 48 = 0$  using factorization method.

**Activity :**

$$x^2 + 8x - 48 = 0$$

$$\therefore x^2 + \boxed{\phantom{00}}x - 4x - 48 = 0$$

$$\therefore x(x + 12) - 4(x + 12) = 0$$

$$\therefore (x + 12) \left( \boxed{\phantom{00}} \right) = 0$$

$$\therefore \boxed{\phantom{00}} = 0 \text{ किंवा } x - 4 = 0$$

$$\therefore x = \boxed{\phantom{00}} \text{ किंवा } x = 4$$

3) Determine the nature of roots of the quadratic equation.

$$3x^2 + 5x - 4 = 0$$

4)  $\alpha$  and  $\beta$  are roots of quadratic equation  $2x^2 + 6x - 5 = 0$ . Find  $\alpha + \beta$  and  $\alpha\beta$ .

5) Obtain the quadratic equation if roots are  $-7$  and  $5$ .

6) Solve the following quadratic equations by factorization.

a)  $3x^2 - x - 10 = 0$       b)  $9m^2 - 16 = 0$       c)  $3y^2 = 15y$

Q. 3) -

- 1) If  $x = 5$  is root of equation  $kx^2 - 14x - 5 = 0$  then find the value of  $k$  by completing the following activity.

Activity :

Take  $x = 5$

$$\therefore kx^2 - 14x - 5 = 0$$

$$\therefore k \boxed{\phantom{00}}^2 - 14 \times \boxed{\phantom{00}} - 5 = 0$$

$$\therefore \boxed{\phantom{00}}k - 70 - 5 = 0$$

$$\therefore 25k = \boxed{\phantom{00}}$$

$$\therefore k = \frac{75}{\boxed{\phantom{00}}}$$

$$\therefore k = \boxed{\phantom{00}}$$

- 2) Solve using formula.  $5m^2 - 4m - 2 = 0$

- 3) Solve by completing the square method.  $3y^2 + 7y + 1 = 0$

4) The sum of squares of two consecutive even natural numbers is 244. Find the numbers.

5) The length of a rectangle is greater than its breadth by 2 cm. If area of rectangle is 24 sqcm then find it's length and breadth.

### 3. Arithmetic Progression

**Previous Knowledge :**

**Number Works :**

- 1) Even Numbers : 2, 4, 6, 8, 10, 12, ...  
Consecutive even numbers have a difference of 2.
- 2) Odd Numbers : 1, 3, 5, 7, 9 ... consecutive odd numbers have a difference of 2.

**Divisible numbers :**

- 1) Numbers divisible by 3 : 3, 6, 9, 12, 15, 18, ...
- 2) Numbers divisible by 5 : 5, 10, 15, 20, 25, ...
- 3) Numbers divisible by 8 : 16, 24, 32, 40, ...

**Addition and Subtraction of Signed Numbers :**

Equal signs : Sum it and give the sign of the numbers given in the answer.

Opposite signs : Large number - Small number and give the sign of large.

Number in the answer.

Ex. i)  $17 + 9 = 26$

ii)  $17 - 9 = 8$

iii)  $-17 - 9 = -26$

iv)  $-17 + 9 = -8$

If there is a bracket after the minus sign change the sign of each term in bracket.  
as the bracket is disappear.

Ex.  $18 - (-7) = 18 + 7 = 25$

**Solve equation :** 1)  $\frac{x}{6} = 4$   
 $\therefore x = 6 \times 4$   
 $\therefore x = 24$

2)  $7x = 28$   
 $\therefore x = \frac{28}{7}$   
 $\therefore x = 4$

**Q. 1 A) Select proper alternative for each subquestions.**

1) If the  $n^{\text{th}}$  term of an A.P. is  $2n - 5$  then find  $t_5$ ?

(A) 15      (B) 5      (C) 10      (D) - 5

**Solution :**  $n^{\text{th}}$  term =  $t_n = 2n - 5$

Put  $n = 5$

$t_n = 2 \times 5 - 5$

$$= 10 - 5$$

$$\therefore t_n = 5$$

$\therefore$  Ans. : Option (B)

2) Find the sum of first 10 even natural numbers.

(A) 55      (B) 45      (C) 20      (D) 60

**Solution :** Here  $n = 10$   $t_1 = 1$

$$S_n = \frac{n}{2} (t_1 + t_n)$$

$$\therefore S_{10} = \frac{10}{2} (1 + 10)$$

$$\therefore S_{10} = 5 \times 11 = 55$$

$\therefore$  Ans. : Option (A)

3) First three terms of an A, P are ....., whose first term is 3 and common difference is 4 ?

(A) 3, 7, 11, ...      (B) 3, -7, 11 ...

(C) 3, -1, -3, ...      (D) 3, -7, -11 ...

4) Find the value of **a** for the arithmetic progression 6, 10, 14, 18, .....

(A) 4      (B) 10      (C) 6      (D) 18

**Q. 1 B) Solve following subquestions.**

1) What is the common difference of these arithmetic progression 2, 4, 6, 8, .....

**Solution :**  $t_1 = 2$ ,  $t_2 = 4$

$$\therefore d = t_2 - t_1$$

$$\therefore = 4 - 2$$

$$\therefore d = 2$$

Common difference = 2

2) If the  $n^{\text{th}}$  term of an A. P. is  $3n + 1$  then find the  $7^{\text{th}}$  term.

3) Find the first term and common difference of the A.P. 3, 5, 7, 9, 11, ...



- 4) Decide whether the sequence 1, 3, 6, 10, ... is an arithmetic progression.
- 5) Find first three terms of an A.P., whose first term is 9 and common difference is 5.

**Q. 2 A) Complete the following activity.**

- 1) Find the sum of first 20 even natural numbers.

Activity : Lets 2, 4, 6, 8, 10, ..... are even natural numbers.

$$\therefore a = 2, d = 4 - 2 = 2, n = 20$$

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$\therefore S_{20} = \frac{20}{2} [2 \times 2 + (20 - 1) \times 2]$$

$$\therefore = 10 (4 + 38) \therefore S_{20} = 420$$

$\therefore$  Sum of first 20 even natural numbers is 420.

- 2) 5, 8, 11, 14, ... In this A.P. which term is number 68

Activity : Here  $a = 5$ ,  $d = \square$ ,  $t_n = 68$

$$t_n = a + (n - 1) d$$

$$\therefore 68 = 5 + (n - 1) \square$$

$$\therefore 68 = 5 + 3n - \square$$

$$\therefore n = \square$$

- 3) In an A.P.  $t_1 = 1$  and  $t_n = 149$  find  $S_n$ ?

Activity : Here  $t_1 = 1$ ,  $t_n = 149$ ,  $S_n = ?$

$$S_n = \frac{n}{2} (t_1 + t_n)$$

$$\therefore S_n = \frac{n}{2} (\square + \square)$$

$$\therefore = \frac{n}{2} \times \square$$

$$S_n = n \square$$

**Q. 2 B) Solve the following subquestions.**

- 1) Find the 11<sup>th</sup> term of the A.P. 3, 9, 15, 21, ...

**Solution :** Given A.P. 3, 9, 15, 21, ...

Here  $a = 3, d = 9 - 3 = 6$

$$t_n = a + (n - 1) d$$

$$\therefore t_{11} = 3 + (11 - 1) \times 6$$

$$\therefore \quad = 3 + 10 \times 6$$

$$\therefore t_{11} = 63$$

- 2) Find how many two digit natural numbers are divisible by 6?

**Solution :** List of two digit natural numbers are divisible by 6 are 6, 12, 16, 20, 24, ...96.

$$t_n = 96, a = 12, d = 6, n = ?$$

$$t_n = a + (n - 1) d$$

$$\therefore 96 = 12 + (n - 1) 6$$

$$\therefore 12 + 6n - 6$$

$$\therefore 6n = 90$$

$$\therefore n = 15$$

$\therefore$  Two digit natural numbers are divisible by 6 is 15.

- 3) In an A.P.  $a = 7$  and  $d = 3$  then find  $S_{10}$ .
- 4) In an A.P. first term is 3 and common difference is 5 then find  $t_{30}$ .
- 5) Find the 18<sup>th</sup> term of the A.P. 1, 7, 13, 19, ...
- 6) Find how many two digit natural numbers are divisible by 5.

**Q. 3 A) Complete the following activity.**

- 1) Kalpana saves some amount every month. In first three months She saves Rs. 100, Rs. 150 and Rs. 200 respectively. In which month will she save Rs. 1200?

**Activity :** Monthly savings of Kalpana is Rs. 100, Rs. 150, Rs. 200 ...Rs. 1200

Here  $d = \text{Rs. } 50$  hence this sequence is A.P.

$$a = 100, \quad d = 50, \quad t_n = 1200 \quad \boxed{\quad}$$

$$t_n = a + (n - 1)$$

$$\therefore 1200 = \frac{1200}{50} + (n - 1) \times 50$$

$$\therefore \boxed{1200} = 50n - \boxed{50}$$

$$\therefore \boxed{100} = n$$

$$\therefore \frac{1200}{50} = n$$

$$\therefore n = \boxed{24}$$

So Rs. 1200 savings will be made in 24 months.

- 2) Find the sum of natural numbers between 1 and 140 which are divisible by 4.

**Activity :** Natural numbers between 1 and 140 which are divisible by 4 are 4, 8, 12, 16, ..... 136.

Here  $d = 4$  hence this sequence is A.P.

$$a = 4, d = 4, t_n = 136, S_n = ?$$

$$t_n = a + (n - 1) d$$

$$\therefore = 4 + (n - 1) \times 4$$

$$\therefore \boxed{\phantom{000}} = (n - 1) \times 4$$

$$\therefore n = \boxed{\phantom{000}}$$

$$\text{Now } S_n = \frac{n}{2} + [a + t_n]$$

$$\therefore S_{34} = 17 \times \boxed{\phantom{000}}$$

$$\therefore S_{34} = \boxed{\phantom{000}}$$

So sum of numbers from 1 to 140, which are divisible by 4.

**Q. 3 B) Solve following subquestions.**

- 1) In the year 2010 in the village there were 5000 people who were literate. Every year the number of literate people increases by 300. How many people will be literate in the year 2020?

**Solution :** From given information  $t_1 = 5000, t_2 = 5300, t_3 = 5600$

$$\therefore a = 5000, d = 300, n = 11$$

$$t_n = a + (n - 1) d$$

$$\therefore = 5000 + (11 - 1) 300$$

$$\therefore = 5000 + 3000$$

$$\therefore t_n = 8000$$

$\therefore$  In year 2020, 8000 people will be literate.

- 2) 5, 9, 13, 17, ... In this A.P. which term is number 41?
- 3) Vijay invested in a national saving certificate scheme. In the first year he invested Rs. 500, in the second year Rs. 700, in the third year Rs. 900 and so on. Find the total amount that he invested in 12 years.
- 4) Find the sum of natural numbers between 1 and 700 which are divisible by 7.
- 5) There is an auditorium with 30 rows of seats. There are 20 seats in the first row, 24 seats in the second row, 28 seats in the third row and so on. Find how many total seats are there in the auditorium?



## 4. Financial Planning

- GST stands for goods and service tax.
- GST is in effect from 1st of July, 2017
- **Components of GST :** (1) CGST (2) SGST
- **CGST :** Central Goods and Service Tax  
**SGST :** State Goods and service Tax  
**CGST = SGST**
- **GSTIN :** Goods and Service Tax Identification Number. GSTIN has 15 is alpha numerals.

### **Rate of GST :**

- It varies with products (Goods)  
e.g, 0%, 5%, 12%, 18% and 28% .
- Rate of GST on brokerage is 18% .

### **Q. 1 A) Write correct alternative for the following questions.**

- 1) GST system was introduced in our country from .....
- A) 31<sup>st</sup> March, 2017      B) 1<sup>st</sup> April, 2017  
C) 1<sup>st</sup> January, 2017      D) 1<sup>st</sup> July, 2017

### **Solution : (D)**

- 2) Rate of GST on essential commodities is .....
- A) 5%      B) 12%      C) 0%      D) 18%

### **Solution : (C)**

- 3) In the format of GSTIN there are ..... alpha numerals.  
A) 15%      B) 10%      C) 16%      D) 9%
- 4) Rate of GST on brokerage is .....

### **B) Solve Following subquestions**

- 1) 'Pawan Medical' supplies medicines. On some medicines the rate of GST is 12% then what is the rate of CGST and SGST?

**Solution :** Rate of CGST and SGST is same.

$$\therefore \text{CGST} = 6\%$$

$$\text{SGST} = 6\%$$

2) On a certain article if rate of CGST is 9% then what is the rate of SGST?

**Solution :** Rate of CGST is 9%

Rate of CGST is 9%

3) Rate of CGST on AC is 14%. Then find rate of SGST and rate of GST.

4) Rate of GST on washing machine is 28%. Then find SGST and CGST.

**Q. 2 A) Complete following activity and rewrite it.**

1) The taxable value of a wrist watch belt is ₹ 586. Rate of GST is 18% Then what is price of belt for the customer?

**Solution :** The taxable value of a wrist watch belt = ₹ 586

Rate of GST = 18%

GST = taxable value × Rate of GST

$$= 586 \times \boxed{18\%}$$

$$= 586 \times \frac{18}{100}$$

$$\text{GST} = \boxed{105.48}$$

Price of belt = taxable value of GST

$$= 586 + \boxed{105.48}$$

$$= ₹ \boxed{691.48}$$

- **Output tax :** The collected GST at the time of sale is called Output tax.
- **Input tax :** The paid GST at the time of purchase is called input tax.
- **Input tax credit :** At the time of paying GST a trader deducts the input tax from the output tax and pays the remaining. This deduction of Input tax is called Input tax credit.

$$\boxed{\text{GST Payable} = \text{Output tax} - \text{ITC}}$$

**Example :**

1) 'Chetana store' paid total GST of ₹ 1,00,500 at the time of purchase and collected GST ₹ 12,2500 at the time of sale. Find the GST payable by Chetana Stores.

**Solution :** GST Payable = Output tax – ITC

$$= 12,2500 - 1,00,500$$

$$= ₹ 22000$$

2) Nazama paid GST of 12,500 on purchase and collected ₹ 14,750 on sale. What is the amount of GST payable ?

- **Share** : A share is the smallest unit of capital.
- **Face value (FV)** : The value printed on share certificate is called face value of the share.
- **Market value (MV)** : The price at which the sold or purchased in the stock market value of the share.
- If  $MV > FV$  then the share is at premium.
- If  $MV = FV$  then the share is at par.
- If  $MV < FV$  then the share is at discount.
- $\text{Sum Invested} = \text{number of shares} \times MV$

**Q. 3 A) Solve following subquestions**

1) If 50 shares of FV ₹ 500 each are purchased for MV ₹ 120. Find the sum invested.

**Solution :**

$$\begin{aligned} \text{sum invested} &= \text{number of shares} \times MV \\ &= 50 \times 120 \\ &= ₹ 6000 \end{aligned}$$

2) FV = ₹ 100, premium = ₹ 65 then MV = ?

**B) Complete the following activity and rewrite it.**

Sr. No.	FV	Share is at premium of - 7	MV
1)	₹ 10	premium of ₹ 7	
2)	₹ 25		₹ 16
3)		at par	₹ 5
4)	₹ 20	premium of ₹ 3	

- **Mutual Fund (MF)** : Mutual fund is professionally managed investment scheme, usually run by an A.M.C.
- **Units** : as we get shares for the investment in sharemarket. We get 'Units' when we invest in mutual fund.
- **Net asset value (NAV)** : The market value of a unit is called 'NAV'

$$\text{Total fund value} = \text{NAV of one unit} \times \text{Number of units}$$

C) -

- 1) If the total value of the mutual fund scheme is ₹ 200 crores and 8 crore units are issued then find the NAV of one unit.

**Solution :** 
$$\text{NAV} = \frac{\text{₹ 200 Crores}}{\text{₹ 8 Crores}}$$
$$= \text{₹ 25}$$

- 2) If NAV of one unit is ₹ 25, then how many units will be allotted for the investment of ₹10,000?





## 5. Probability

**Let's Recall :**

**1) Random Experiment :**

In random experiment all possible results are known in advance but none of them can be predicted with certainty. There is equal possibility for each result.

**2) Outcome :**

Result of a random experiment is known as an outcome.

**3) Sample Space :**

The set of all possible outcomes of a random experiment is called the sample space. It is denoted by 'S' or  $\Omega$ . The number of elements in the set 'S' is denoted by  $n(S)$ .

**4) Event :**

A set of favourable outcomes of a given sample space is an 'event'.

Events are generally denoted by capital letters A, B, C, D etc.

The number of elements in event 'A' is denoted as  $n(A)$ .

**5) Probability of an event :**

For a random experiment, if sample space 'S' and 'A' is an expected event then probability of A is  $P(A)$ . It is given by the formula.

$$P(A) = \frac{\text{Number of sample points in an event A}}{\text{Number of sample points in sample spaces}} = \frac{n(A)}{n(S)}$$

**6) The probability of any event is from 0 to 1 or 0% to 100%.**

If A is any event then  $0 \leq P(A) \leq 1$  or  $0\% \leq P(A) \leq 100\%$

**Q. 1 A) Choose the correct alternative answer for each of the following questions.**

1) If a coin is tossed then number of sample points in sample space are ...

- A) 3                      B) 4                      C) 2                      D) 8

**Explanation :** A coin is tossed

$$S = \{H, T\}$$

$$\therefore n(S) = 2$$

$\therefore$  Answer : Option (C) 2

2) If  $n(A) = 2$ ,  $p(A) = \frac{1}{5}$  तब  $n(S) = ?$

A) 10      B)  $\frac{5}{2}$       C)  $\frac{2}{3}$       D)  $\frac{1}{3}$

**Explanation :**  $p(A) = \frac{n(A)}{n(S)}$

$$\therefore \frac{1}{5} = \frac{2}{n(S)}$$

$$\therefore 1 \times n(S) = 2 \times 5$$

$$\therefore n(S) = 10$$

$\therefore$  Answer : Option (A) 10

**For Practice :**

1) Which number can not represent a probability?

A)  $\frac{2}{3}$       B) 1.5      C) 15 %      D) 0.7

2) A die is rolled. Then probability that the number appearing on upper face less than 3 is ...

A)  $\frac{1}{6}$       B)  $\frac{1}{3}$       C)  $\frac{1}{2}$       D) 0

**Q. 1 B) Solve the following subquestions :**

1) One die is rolled then write sample spaces and number of sample points  $n(s)$ .

Answer : One die is rolled.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\therefore n(S) = 6$$

2) If  $n(A) = 12$  and  $n(S) = 52$  then find  $P(A)$ .

Answer :  $P(A) = \frac{n(A)}{n(S)}$

$$\therefore P(A) = \frac{12}{52}$$

$$\therefore P(A) = \frac{3}{13}$$

**For Practice :**

1) Two coins are tossed then write sample space 'S' and number of sample points 'n(S)'.

- 2) There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Write sample space 'S' and number of sample points 'n(S)'.

**Q. 2 A) Complete the following activity.**

- 1) Two coins are tossed simultaneously. The condition for event 'A' is getting no head. Complete the following activity to write S, n(S), A, n(A)

**Activity : Two coins are tossed simultaneously.**

$$S = \boxed{\{HH, HT, TH, TT\}}$$

$$n(S) = \boxed{4}$$

Condition for event A : getting no head.

$$A = \boxed{\{TT\}}$$

$$n(A) = \boxed{1}$$

**For Practice :**

- 1) If a coin is tossed, the condition for event 'A' is getting a head then complete the following activity to find the probability of getting a head.

**Activity :** A coin is tossed.

$$\therefore S = \boxed{\phantom{\{H, T\}}}$$

$$\therefore n(S) = 2$$

Condition for event A : getting a head.

$$A = \boxed{\phantom{\{H\}}}$$

$$n(A) = 1$$

$$\text{Now, } \boxed{\phantom{0.5}}$$

$$p(A) = \frac{\boxed{\phantom{1}}}{n(S)}$$

$$\therefore p(A) = \boxed{\phantom{0.5}}$$

- 2) A card is drawn from a well shuffled pack of 52 playing cards. Complete the following activity to find the probability that the card drawn is a red card.

**Activity :** Let 'S' be the sample space.

$$\therefore D(S) = \boxed{\phantom{52}}$$

$$\text{Total red cards, } n(A) = \boxed{\phantom{26}}$$

$$p(A) = \frac{n(A)}{\boxed{\phantom{52}}}$$

$$p(A) = \boxed{\phantom{0.5}}$$

$$\frac{\boxed{\phantom{1}}}{n(S)}$$

**Q. 2 B) Solve the following subquestions :**

- 1) A die is rolled. The condition for event 'A' is getting even number on upper face.

Write  $S$ ,  $n(S)$ ,  $A$ ,  $n(A)$

Answer : A die is rolled.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\therefore n(S) = 6$$

Event A : getting even number on upper face.

$$A = \{2, 4, 6\}$$

$$\therefore n(A) = 3$$

**For Practice :**

- 1) One coin and a die are thrown simultaneously. The condition for an event 'A' is to get head and an add number. Write  $S$ ,  $n(S)$ ,  $A$ ,  $n(A)$ .
- 2) A sanitation committee of 2 members is to be formed from 3 boys and 2 girls. Write sample space 'S' and the number of sample points  $n(S)$ .

**Q. 3 A) Complete the following Activity :**

- 1) If one die is rolled then complete the following activity to find the probability of an event that the number on upper face is prime.

**Activity :** One die is rolled.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$n(S) = 6$$

Let event A : Prime number on upper face.

$$A = \{2, 3, 5\}$$

$$n(A) = 3$$

$$p(A) = \frac{n(A)}{n(S)}$$

$$p(A) = \frac{3}{6}$$

$$p(A) = \square$$

**For Practice :**

- 1) A box contains 5 strawberry chocolates, 6 coffee chocolates and 2 peppermint chocolates. Complete the following activity to find the probability of each of the following events, if one of the chocolates is picked from the box at random (i) It is a coffee chocolate, (ii) It is a peppermint chocolate.

**Activity :** Let 'S' is sample space.

$$\therefore n(S) = \square$$

Event A : It is a coffee chocolate.

$$\therefore n(A) = \square$$

$$\therefore p(A) = \frac{n(A)}{\square}$$

$$\therefore p(A) = \frac{\square}{\square}$$

$$\therefore p(A) = \frac{\square}{\square}$$

Event B : It is a coffee chocolate.

$$\therefore n(B) = \square$$

$$p(B) = \frac{n(B)}{n(S)}$$

$$p(B) = \frac{\square}{13}$$

**Q. 3 B) Solve the following subquestions :**

- 1) Three coins are tossed simultaneously
- 2) Condition for event A : To get at least two heads.  
Condition for event B : To get head on the second coin. Write S, n(S), A, n(A), B, n(B).

**Answer :** Three coins are tossed simultaneously.

$$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$$

$$\therefore n(S) = 8$$

Condition for event A : To get atleast two heads.

$$A = \{HHH, HHT, HTH, THH, \}$$

$$n(A) = 4$$

Condition for event B : To get head on the second coin.

$$B = \{HHH, HHT, THH, THT\}$$

$$\therefore n(B) = 4$$

**For Practice :**

- 1) A two digit number is formed with digits 2, 3, 5, 7, 9, without repetition. What is the probability that the number formed is ...
  - (i) an odd number.
  - (ii) a multiple of 5.
  
- 2) A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is ...
  - (i) an ace.
  - (ii) a spade.
  
- 3) A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets, ...
  - (i) a red balloon
  - (ii) a blue balloon.



## 6. Statistics

• **Measures of central tendency :**

A) Mean B) Median C) Mode

**A) Mean :** The arithmetical average of all observation in the given data is known as its arithmetic mean or simply mean.

**Methods of calculating Mean**

**1) Direct Method :** Frequency distribution table for direct method is as follows :

Class	Class mark ( $x_i$ )	Frequency ( $f_i$ )	Class mark $\times$ Frequency $x_i f_i$
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**Formula :**

$$\text{Mean} = \bar{X} = \frac{\sum x_i f_i}{N}$$

Where  $\sum x_i f_i$  = sum of  $x_i f_i$      $N$  = Total frequencies

**2) Assumed Mean Method :** Frequency distribution table for assumed mean method is as follows.

Class	Class mark ( $x_i$ )	$d_i = x_i - A$	Frequency ( $f_i$ )	Class mark $\times$ Frequency $f_i d_i$
-------	-------------------------	-----------------	------------------------	--------------------------------------------

**Formula :**

$$\text{Mean} = \bar{X} = A + \bar{d}$$

Where  $A_i$  = Assumed Mean. (You can take any class mark as A but to make calculation simple, take middle class mark as A)

$$\bar{d} = \frac{\sum f_i d_i}{N}$$

$\sum f_i d_i$  = Sum of  $f_i d_i$      $N$  = Total frequencies

**3) Step Deviation Method :** Frequency distribution table for step deviation method is as follows :

Class	Class mark ( $x_i$ )	$d_i = x_i - A$	$u_i = \frac{d_i}{g}$	Frequency ( $f_i$ )	$f_i u_i$
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**Formula :**

$$\text{Mean} = \bar{X} = A + g \bar{u}$$

Where  $A$  = Assumed Mean

$g$  = H.C.F (G.C.D) of all values of  $d_i$

$$\bar{u}_i = \frac{\sum f_i u_i}{N}$$

**B) Median :** The point which divides given data exactly in two equal parts is known as Median. In short median is midpoint of data.

To calculate median, classes have to be continuous, if not make them continuous.

Frequency distribution table for calculating Median as follows :

Class	Continuous classes (If given class is not continuous)	Frequency ( $f_i$ )	Cumulative frequency (less than type)
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**Formula :**

$$\text{Median} = L + \left[ \frac{\frac{N}{2} - cf}{f} \right] \times h$$

Where,

$L$  = Lower class limit of median class.

$h$  = Class interval of median class.

$cf$  = The frequency of the class preceding the median class.

$f$  = The frequency of median class.

$N$  = Total frequency

**Median Class :** Class in which half or more than half  $\left(\frac{N}{2}\right)$  of less than type cumulative frequency appears first is called Median class.

**C) Mode :** The score repeating maximum number of times in a data is called the mode of the data.

To calculate mode, classes have to be continuous if not, make them continuous.

Frequency distribution table to calculate mode is as follows :

Class	Continuous classes (If given class is not continuous)	Frequency ( $f_i$ )
-------	----------------------------------------------------------	------------------------

**Formula :**

$$\text{Mode} = L + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$



Where,

$L$  = Lower class limit of the modal class.

$f_1$  = Frequency of the modal class.

$f_0$  = Frequency of the class preceding the modal class.

$f_2$  = Frequency of the class succeeding the modal class.

$h$  = Class interval of the modal class.

**Modal class** : Modal class is the class which have highest frequency in distribution table.

• **Graphical representation of data :**

A) Histogram   B) Frequency Polygon

**A) Histogram :**

Methods of drawing a histogram :

- 1) If the given classes are not continuous make them continuous.
- 2) Show the classes on the X-axis with a proper scale. (In general class interval is the proper scale)
- 3) Show the frequencies of the Y-axis with a proper scale.
- 4) Taking each class as base, draw rectangles with height proportional to the frequencies of the respective classes.
- 5) If there is no observation upto the first class we use  $\neg$  (krink mark) on X-axis and it is shown between origin and the first class.

**B) Frequency Polygon :**

There are two methods of drawing a frequency polygon.

- 1) With the help of a histogram.
- 2) Without the help of a histogram.

**1) Drawing frequency polygon with the help of histogram :**

- a) Draw histogram.
- b) Mark the midpoint of upper side of each rectangle in the histogram.
- c) Assume that a rectangle of zero height exists preceding the first rectangle and mark its midpoint. Similarly assume a rectangle of zero height succeeding the last rectangle and mark its mid point.
- d) Join all mid point in order by the line segments.

**2) Drawing frequency polygon without the help of histogram :**

- a) If the given classes are not continuous, make them continuous.
- b) Show the classes on the X-axis with a proper scale.
- c) Show the frequencies of the Y-axis with a proper scale.
- d) Observe the following table. It shows how the co-ordinates of points are decided to draw frequency polygon.

Class	Continuous classes	Class Mark	Frequency	Co-ordinates of point
6 - 7	5.5 -7.5	6.5	0	(6.5, 0)
8 - 9	7.5 -9.5	8.5	20	(8.5, 20)
10 - 11	9.5 -11.5	10.5	40	(10.5, 40)
12 - 13	11.5 -13.5	12.5	30	(12.5, 30)
14 - 15	13.5 -15.5	14.5	15	(14.5, 15)
16 - 17	15.5 -17.5	16.5	0	(16.5, 0)

The points corresponding to the co-ordinates in the fifth column are plotted. Joining them in order by line segment, we get a frequency polygon.

**• Pictorial Representation of data :**

**Pie diagram :**

- 1) Every component of a data is shown by a sector associated with it.
- 2) The measure of the central angle of the sector is in proportion with the number of scores in that component.

The measure of central angle of each sector is found by the following formula.

$$\text{The measure of central angle} = \theta = \frac{\text{No. of scores in the component}}{\text{Total No. of scores}} \times 360$$

- 3) A circle of a suitable radius is drawn. Then it is divided into sectors such that the number of sectors is equal to the number of components in the data.

**Questions :**

**Q. 1 A) Choose the correct answer and write the alphabet of it in front of the sub-question number.**

- 1) Cumulative frequencies in a grouped frequency table are useful to find .....  
A) Mean      B) Median      C) Mode      D) All of these

**Solution : B**

2) The formula to find mean from a grouped frequency table is

$$\bar{X} = A + \frac{\sum f_i u_i}{\sum f_i} \times hg. \text{ In the formula } u_i = ?$$

- A)  $\frac{x_i + A}{g}$       B)  $(x_i - A)$       C)  $\frac{x_i - A}{g}$       D)  $\frac{A - x_i}{g}$

**Solution :** C

**Examples for practice**

1) Which of the following is the score repeating maximum number of times in a data?  
 A) Mean      B) Median      C) Mode      D) Average

2)

<b>No. of trees planted by each student</b>	1 - 3	4 - 6	7 - 9	10 - 12
<b>No. of students</b>	7	8	6	4

the above data is to be shown by a frequency polygon. The co-ordinates of the point to show number of students in the class 4 - 6 are .....

- A) (4, 8)      B) (6, 8)      C) (5, 8)      D) (8, 4)

3) Which of the following is graphical representation of data?

- A) bar graph      B) histogram      C) pie diagram      D) all of the above

**Q. 1 B) Solve the following sub-questions.**

1) Find the class mark of the class 20 - 24.

**Solution :**

$$\text{Class mark} = \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

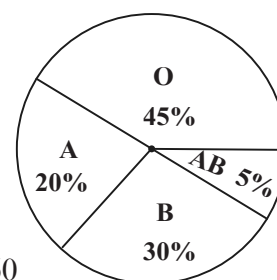
$$= \frac{20 + 24}{2}$$

$$= \frac{44}{2}$$

$$= 22$$

∴ Class mark of the class 20 - 24 is 22.

- 2) The pie-diagram shown percentage of persons according to the blood group. Answer the following question. Find the measure of central angle for blood group A.



**Solution :**

$$\begin{aligned} \text{Measure of central angle of blood group A} &= \frac{\text{Percentage of persons of blood group A}}{\text{Total percentage}} \times 360 \\ &= \frac{5}{100} \times 360 \\ &= 18^\circ \end{aligned}$$

### Examples for practice

- 1) If  $\sum f_i x_i = 75$  and  $\sum f_i = 15$  then find the mean  $\bar{X} = ?$
- 2) Find the width of class 35 - 45.
- 3) If  $\sum f_i d_i = 108$  and  $\sum f_i = 100$  then find  $\bar{d} = ?$

**Q. 2 A) Complete any two given activities and rewrite it.**

- 1) The following table shows the daily supply of electricity to different places in a town. To show the information by pie diagram measures of central angles of sectors are to be decided. Complete the following activity to find the measures.

Places	Supply of electricity (Thousand units)	Measure of central angle
Roads	4	$\frac{4}{30} \times 360 = 48^\circ$
Factories	12	$\frac{\quad}{\quad} \times 360 = 144^\circ$
Shops	6	$\frac{6}{30} \times 360 = \square$
Houses	8	$\frac{\quad}{\quad} \times 360 = \square$
<b>Total</b>	30	

**Solution :**

Places	Supply of electricity (Thousand units)	Measure of central angle
Roads	4	$\frac{4}{30} \times 360 = 48^\circ$
Factories	12	$\frac{12}{30} \times 360 = 144^\circ$
Shops	6	$\frac{6}{30} \times 360 = 72^\circ$
Houses	8	$\frac{8}{30} \times 360 = 96^\circ$
<b>Total</b>	30	

- 2) Complete the activity to prepare a table showing the co-ordinates which are necessary to draw frequency polygon.

Class	18 - 19	19 - 20	20 - 21	<input type="text"/>
Class mark	18.5	19.5	<input type="text"/>	21.5
Frequency	4	<input type="text"/>	15	19
Co-ordinates of point	<input type="text"/>	(19.5,13)	(20.5, 15)	(21.5,19)

**Solution :**

Class	18-19	19-20	20-21	<b>21-22</b>
Class mark	18.5	19.5	<b>20.5</b>	21.5
Frequency	4	<b>13</b>	15	19
Co-ordinates of point	<b>18.5,4</b>	(19.5,13)	(20.5, 15)	(21.5,19)

**Examples for practice**

- 1) Complete the activity to prepare the cumulative frequency (less than type) table from the given frequency distribution.

Class	Frequency	Cumulative Frequency (less than type)
0 - 10	4	4
10 - 20	8	<input type="text"/>
<input type="text"/>	10	22
30 - 40	<input type="text"/>	27
40 - 50	3	<input type="text"/>

- 2) Below is the distribution of money (in ₹) collected by 25 students for flood relief fund. Complete the table to find mean of money (in ₹) collected by a student using direct method.

Money (in ₹)	Class mark ( $x_i$ )	No. of students ( $f_i$ )	$x_i f_i$
0-10	5	5	25
<input type="text"/>	15	<input type="text"/>	30
20-30	<input type="text"/>	6	150
30-40	35	5	175
40-50	45	7	<input type="text"/>

- 3) Area under different crops in a certain village is given below. To show the information by pre-diagram, measures of central angles of sectors are to be decided. Complete the following activity to find the measures.

Crop	Area in hectares	Measure of central angle
Jowar	40	$\frac{\quad}{\quad} \times 360 = 80^\circ$
Wheat	60	$\frac{60}{180} \times 360 = 120^\circ$
Sugarcane	50	$\frac{50}{180} \times 360 = \square$
Vegetable	30	$\frac{\quad}{\quad} \times 360 = \square$
<b>Total</b>	<b>180</b>	

**Q. 2 B) Solve the following sub-questions. (Any four)**

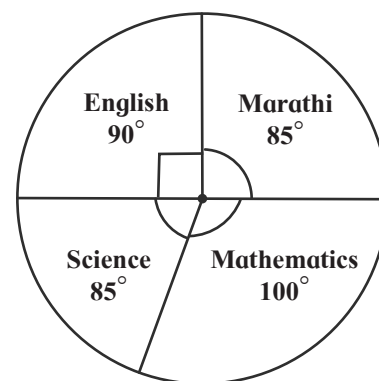
- 1) Subjectwise marks obtained by a student in an examination are given below.

Subject	Marathi	English	Science	Mathematics	Total
Marks	85	90	85	100	360

Draw a pie-diagram to represent the above data.

**Solution :**

Subject	Marks	Measure of central angle
Marathi	85	$\frac{85}{360} \times 360 = 85^\circ$
English	90	$\frac{90}{360} \times 360 = 90^\circ$
Science	85	$\frac{85}{360} \times 360 = 85^\circ$
Mathematics	100	$\frac{100}{360} \times 360 = 100^\circ$
<b>Total</b>	<b>360</b>	



2) Below is the given frequency distribution of words in an essay.

<b>Number of words</b>	600-800	800-1000	1000-1200	1200-1400	1400-1600
<b>Number of candidates</b>	14	22	30	18	16

**Solution :**

Number of words	Class mark ( $x_i$ )	$d_i = x_i - A$ $= x_i - 1100$	Number of candidates ( $f_i$ )	$f_i d_i$
600-800	700	- 400	14	- 5600
800-1000	900	- 200	22	- 4400
1000-1200	1100 - A	0	30	0
1200-1400	1300	200	18	3600
1400-1600	1500	400	16	6400
			$\Sigma f_i = N = 100$	$\Sigma f_i d_i = 0$

$$\text{Now, } \bar{d} = \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= \frac{0}{100}$$

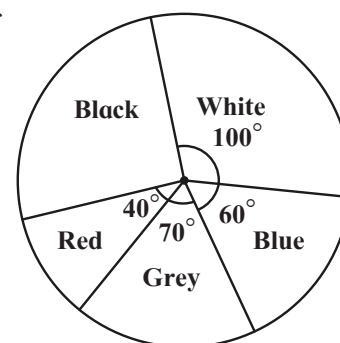
$$= 0$$

$$\begin{aligned} \therefore \text{Mean number of words} &= \bar{X} = A + \bar{d} \\ &= 1100 + 0 \\ &= 1100 \end{aligned}$$

### Examples for practice

1) In a bicycle shop, number of bicycles purchased and choice of their colour was shown in following pie diagram. If in all 36 bicycles were purchased, then answer the following questions with its help.

- 1) Find the central angle for black coloured bicycle.
- 2) Find the number of black coloured bicycle purchased.



2) Find the mean from the given values.

$$\Sigma x_i f_i = 1265, N = 50$$



3) Observe the following table and find mean.

Assumed mean  $A = 300$

Class	Class mark ( $x_i$ )	$d_i = x_i - A$ $= x_i - 300$	$u_i = \frac{d_i}{h}$	Frequency $f_i$	$f_i \times u_i$
200-240	220	- 80	- 2	5	- 10
240-280	260	- 40	- 1	10	- 10
290-320	<input type="text" value="300 - A"/>	0	0	15	0
320-360	340	40	1	12	12
360-400	380	80	2	8	16
<b>Total</b>				$\Sigma f_i = 50$	$\Sigma f_i u_i = 8$

**Q. 3 B) Complete any one activity and rewrite it.**

1) The following table shows classification of number of workers and the number of hours they work in a software company. Complete the activity to find the median of the number of hours they work.

Daily No. of hours	No. of workers	Cumulative frequency (less than)
8 - 10	150	150
10 - 12	500	<input type="text"/>
12 - 14	300	950
14 - 16	50	<input type="text"/>

Here,

$$L = 10 \quad \frac{N}{2} = \text{input} \quad cf = 150, h = 2, f = \text{input}$$

$$\text{Median} = L + \left[ \frac{\frac{N}{2} - cf}{f} \right] \times h \dots\dots\dots \text{(formula)}$$

$$= 10 + \left[ \frac{500 - 150}{500} \right] \times 2 \dots\dots\dots \text{(Substituting the values)}$$

$$= 10 + \frac{\text{input}}{5}$$

$$= 10 + \boxed{\phantom{00}}$$

$$= 11.4$$

∴ Median of the number of hours they work = 11.4

**Solution :**

Daily No. of hours	No. of workers	Cumulative frequency (less than)
8 - 10	150	150
10 - 12	500	<b>650</b>
12 - 14	300	950
14 - 16	50	<b>1000</b>

Here, Median = L

$$L = 10 \quad \frac{N}{2} = \boxed{500} \quad cf = 150, h = 2, f = \boxed{500}$$

$$\text{Median} = L + \left[ \frac{\frac{N}{2} - cf}{f} \right] \times h \dots\dots\dots \text{(formula)}$$

$$= 10 + \left[ \frac{500 - 150}{500} \right] \times 2 \dots\dots\dots \text{(Substituting the values)}$$

$$= 10 + \frac{\boxed{7}}{5}$$

$$= 10 + \boxed{1.4}$$

$$= 11.4$$

∴ Median of the number of hours they work = 11.4

- 2) The following frequency distribution table shows the classification of the number of vehicles and the volume of petrol filled in them. To find the mode of the volume of petrol filled, complete the following activity.

Class (petrol filled in liters)	Frequency (No. of vehicles)
0.5 - 3.5	33
3.5 - 6.5	40
6.5 - 9.5	27
9.5 - 12.5	18
12.5 - 15.5	12

**Activity :**

From the given table,

$$\text{Modal class} = \boxed{\phantom{0.5 - 3.5}}$$

$$\therefore \text{Mode} = \boxed{\phantom{3.5}} + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - \boxed{\phantom{27}}} \right] \times h \dots\dots\dots \text{(formula)}$$

$$\therefore \text{Mode} = 3.5 + \left[ \frac{40 - 33}{2(40) - 33 - 27} \right] \times \boxed{\phantom{3}} \dots\dots\dots \text{(substituting the values)}$$

$$= 3.5 + \left[ \frac{7}{80 - 60} \right] \times 3$$

$$\therefore \text{Mode} = \boxed{\phantom{6.5}}$$

$\therefore$  The mode of the volume of petrol filled is  $\boxed{\phantom{6.5}}$  liter.

**Solution :**

From the given table,

$$\text{Modal class} = \boxed{3.5 - 6.5}$$

$$\therefore \text{Mode} = \boxed{L} + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - \boxed{f_2}} \right] \times h \dots\dots\dots \text{(formula)}$$

$$\therefore \text{Mode} = 3.5 + \left[ \frac{40 - 33}{2(40) - 33 - 27} \right] \times \boxed{3} \dots\dots\dots \text{(substituting the values)}$$

$$= 3.5 + \left[ \frac{7}{80 - 60} \right] \times 3$$

$$\therefore \text{Mode} = \boxed{4.55}$$

$\therefore$  The mode of the volume of petrol filled is  $\boxed{4.55}$  liter.

### Examples for practice

- 1) The production of electric bulbs in different factories is shown in the following table. To find the median of the productions, complete the following activity.

No. of bulbs produced (thousands)	No. of factories	Cumulative frequency less than
30 - 40	12	12
40 - 50	35	47
50 - 60	20	67
60 - 70	15	82
70 - 80	8	90
80 - 90	7	97
90 - 100	8	105

**Activity :**

From the given table,

Median class =

$$\therefore \text{Median} = L + \left[ \frac{\boxed{\phantom{00}} - cf}{\boxed{\phantom{00}}} \right] \times h \dots\dots\dots \text{(formula)}$$

$$= 50 + \left[ \frac{52.5 - \boxed{\phantom{00}}}{20} \right] \times 10 \dots\dots\dots \text{(substituting the value)}$$

$$= 50 + \frac{55}{20}$$

$$\therefore \text{Median} = \boxed{\phantom{00}}$$

$\therefore$  Median of the production is

- 2) The following table shows the classification of children according to their ages, playing on ground. Complete the activity to find the mode of ages of the children.

<b>Age groups of children (Yrs.)</b>	6 - 8	8-10	10-12	12-14	14-16
<b>No. of children</b>	43	58	70	42	27

**Activity :**

here,

$$L = 10, f_0 = \square, f_1 = 70, f_2 = 42, h = 2$$

$$\therefore \text{Mode} = L + \left[ \frac{f_1 - f_0}{\square - f_0 - f_2} \right] \times h \dots\dots\dots \text{(formula)}$$

$$= 10 + \left[ \frac{70 - 58}{2(70) - 58 - 42} \right] \times 2 \dots\dots\dots \text{(substituting the values)}$$

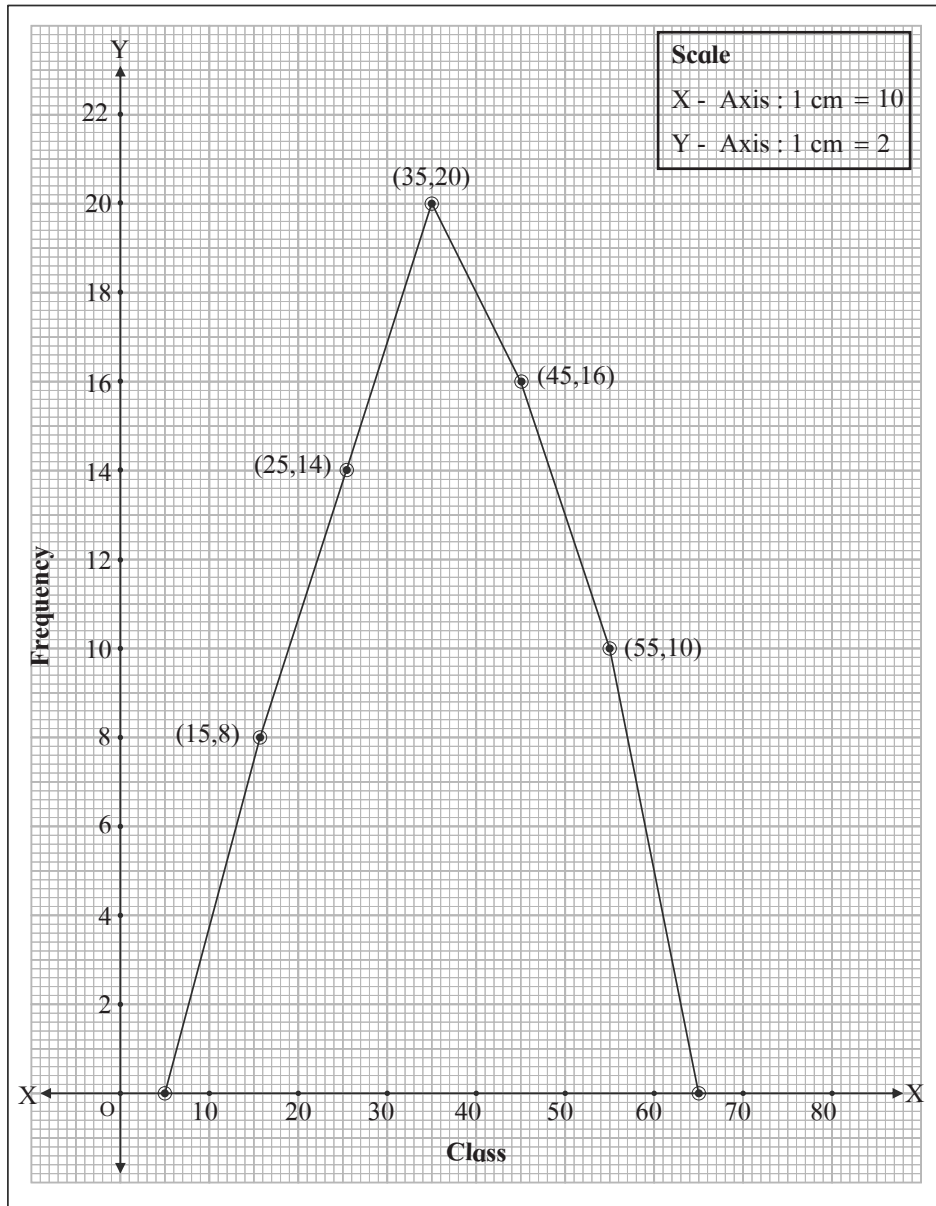
$$\therefore \text{Mode} = 10 + \frac{12}{\square} \times 2$$

$$= 10 + \frac{24}{\square}$$

$$= \square$$

$\therefore$  The mode of the ages of children playing on ground is  $\square$  years.

Q. 3 A) Complete the activity based on frequency polygon given in adjacent figure.



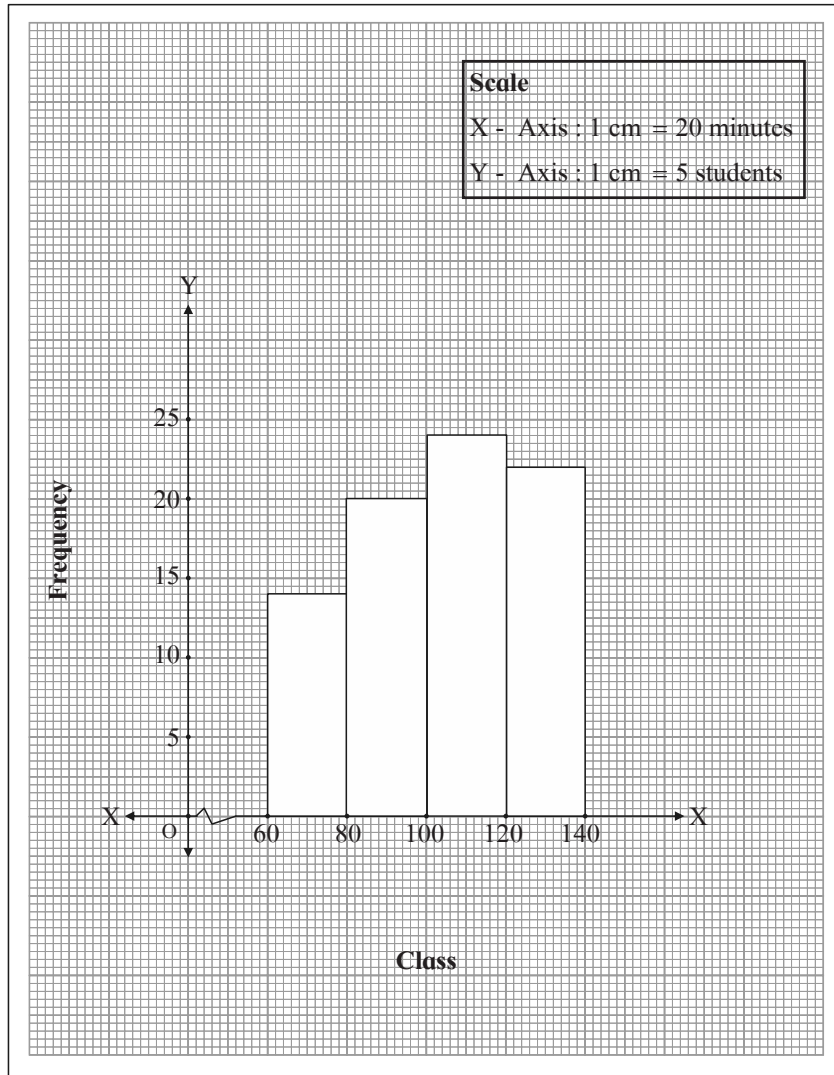
- 1) Write the frequency of the class 50 - 60
- 2) Write the class in which frequency is maximum
- 3) State the class whose class mark is 55
- 4) State the class whose frequency is 14
- 5) Write the class whose frequency is zero
- 6) Write the upper class limit of class whose lower class limit is 30


**Q. 3 b) Solve the following sub-questions. (Any two)**

- 1) Time allotted for the preparation of an examination by some is shown in the table. Draw Histogram to show this information.

<b>Time (minutes)</b>	60 - 80	80 - 100	100 - 120	120 - 140
<b>No. of students</b>	14	20	24	22

**Solution :**

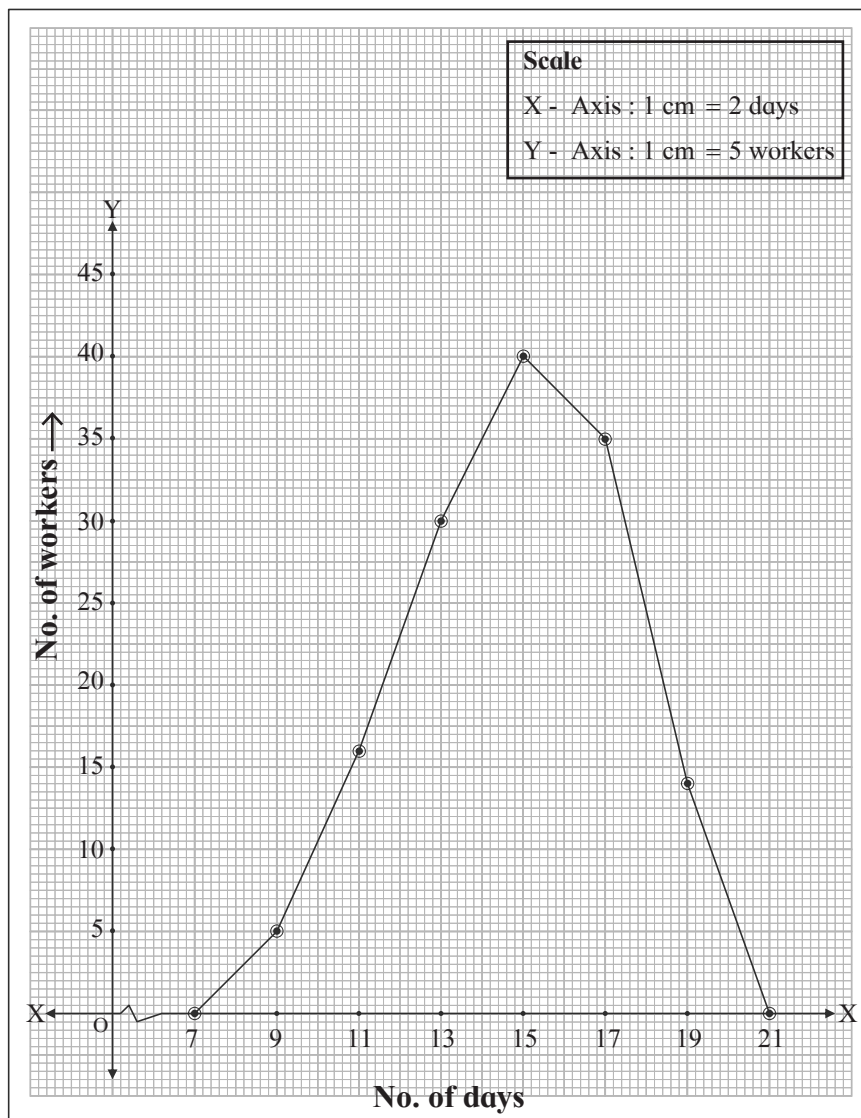


- 2) In a handloom factory different workers take different periods of time to weave a saree. The number of workers and their required periods are given below. Draw a frequency polygon of it.

<b>No. of days</b>	8-10	10-12	12-14	14-16	16-18	18-20
<b>No. of workers</b>	5	16	30	40	35	14

**Solution :**

No. of days	Class mark	No. of workers	Co-ordinates of points
6 - 8	7	0	(7, 0)
8 - 10	9	5	(9, 5)
10 - 12	11	16	(11, 16)
12 - 14	13	30	(13, 30)
14 - 16	15	40	(15, 40)
16 - 18	17	35	(17, 35)
18 - 20	19	14	(19, 14)
20 - 22	21	0	(21, 0)





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**Examples for practice**

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- 1) Draw histogram and hence the frequency polygon for the following frequency distribution.

<b>Rainfall (in cm)</b>	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
<b>No. of villages</b>	2	5	8	12	10	7

- 2) The marks scored by students in mathematics in certain examination are given below.

<b>Marks scored</b>	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
<b>Number of students</b>	3	8	15	17	7

Draw histogram for the above data.

- 3) Show the following data by a frequency polygon.

<b>Electricity bill (₹)</b>	0 - 200	200 - 400	400 - 600	600 - 800	800 - 1000
<b>Families</b>	240	300	450	350	160



## Part - 2

### 1. Similarity

#### Previous Knowledge :

$$1) \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{b_1 \times h_1}{b_2 \times h_2}$$

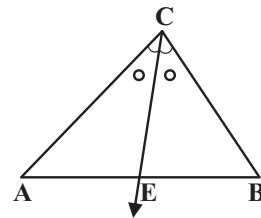
$$2) \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{b_1}{b_2} \quad \text{if } h_1 = h_2$$

$$3) \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{h_1}{h_2} \quad \text{if } b_1 = b_2$$

4) In  $\Delta ABC$ ,  $DE \parallel \text{seg } BC$  then  $\frac{AD}{DC} = \frac{AE}{EC}$  ..... theorem of basic proportionality.

5) In  $\Delta ABC$  bisector of  $\angle C$  intersects  $\text{seg } AB$  in point  $E$ ,

$$\text{then } \frac{AE}{EB} = \frac{CA}{CB}$$



6) If  $\Delta ABC \sim \Delta PQR$  then  $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

7) If  $\Delta_1 \sim \Delta_2$  then  $\frac{A(\Delta_1)}{A(\Delta_2)} = \frac{h_1^2}{h_2^2} = \frac{b_1^2}{b_2^2} = \frac{s_1^2}{s_2^2}$  .....  $h$  - height,  $b$  - base,  $s$  - side

#### Example :

#### Q. 1 A) Choose the correct alternative.

1) The areas of two similar triangles are  $9 \text{ cm}^2$  and  $16 \text{ cm}^2$ . The ratio of their corresponding heights is.....

- (A) 9 : 16      (B) 3 : 4      (C) 4 : 3      (D) 16 : 9

$$\Delta_1 \sim \Delta_2 \text{ then } \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{h_1^2}{h_2^2}$$

$$\therefore \frac{9}{16} = \frac{h_1^2}{h_2^2}$$

$$\therefore \frac{h_1}{h_2} = \frac{3}{4}$$

**Ans :** (B)

- 2) If  $\triangle ABC \sim \triangle PQR$  and  $\angle A = 45^\circ$ ,  $\angle Q = 87^\circ$ , then  $m\angle C = \dots\dots\dots$   
 (A)  $45^\circ$             (B)  $87^\circ$             (C)  $48^\circ$             (D)  $90^\circ$

Using Angle sum property of triangle,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle C = 180^\circ - (45^\circ + 87^\circ)$$

$$\angle C = 48^\circ$$

**Ans :** (C)

- 3) If  $\triangle ABC \sim \triangle DEF$  and  $\angle A = 48^\circ$ , then  $\angle D = \dots\dots\dots$   
 (A)  $48^\circ$             (B)  $83^\circ$             (C)  $49^\circ$             (D)  $132^\circ$

**Q. 1 B) Attempt the following.**

- 1) If  $\triangle ABC \sim \triangle PQR$ ,  $A(\triangle ABC) = 80$ ,  $A(\triangle PQR) = 125$ , then find  $AB : PQ$

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{80}{125} = \frac{AB^2}{PQ^2} \dots\dots(\text{Areas of similar triangle})$$

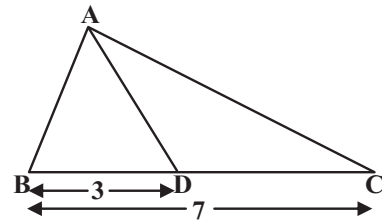
$$\therefore \frac{AB^2}{PQ^2} = \frac{16}{25}$$

$$\therefore \text{Ans : } \frac{AB}{PQ} = \frac{4}{5}$$

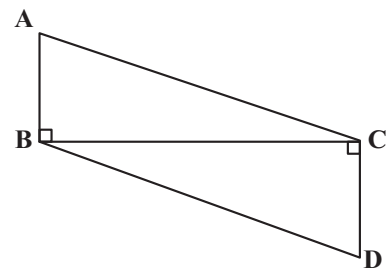
- 2) In the figure  $BC = 7$ ,  $BD = 3$ . Write the ratio  $\frac{A(\Delta ABD)}{A(\Delta ABC)}$

$$\frac{A(\Delta ABD)}{A(\Delta ABC)} = \frac{BD}{BC} \dots\dots \text{(triangles having common heights)}$$

$$\therefore \text{Ans : } \frac{BD}{BC} = \frac{3}{7}$$



- 3) In the given figure.  $seg AB \perp seg BC$  and  $seg DC \perp seg BC$ . If  $AB = 3$  cm and  $CD = 4$  cm, then find  $\frac{A(\Delta ABC)}{A(\Delta DCB)}$



- 4) The ratio of corresponding sides of similar triangles is  $3 : 5$ ; then find the ratio of their areas.

- 5) If  $\Delta ABC \sim \Delta PQR$ , and  $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{16}{25}$  then, find  $AB : PQ$



3) If  $\Delta ABC$  seg  $DE \parallel$  side  $BC$ .

If  $AD = 6$  cm  $DB = 9$  cm seg  $EC = 7.5$  cm

then complete the following activity to find  $AE$ .

**Activity :** In  $\Delta ABC$

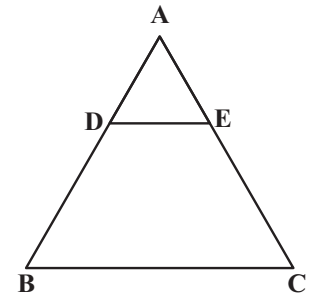
seg  $DE \parallel$  side  $BC$  ..... (given)

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \dots\dots\dots \boxed{\dots\dots\dots}$$

$$\therefore \frac{6}{9} = \frac{AE}{\boxed{\dots\dots\dots}}$$

$$\therefore AE = \frac{6 \times 7.5}{\boxed{\dots\dots\dots}}$$

$$\therefore AE = \boxed{\dots\dots\dots}$$



**Q. 2 B) Attempt the following.**

1)  $\Delta LMN \sim \Delta RST$ ,  $LM = 3$ ,  $MN = 4$ ,  $ST = 12$ , find  $RS$ .

**Ans :** Sides of similar triangles are proportional.

$$\therefore \frac{LM}{RS} = \frac{MN}{ST}$$

$$\therefore \frac{3}{RS} = \frac{4}{12}$$

$$\therefore RS = \frac{3 \times 12}{4}$$

$$\therefore RS = 9 \text{ units}$$

2)  $\Delta LMN \sim \Delta PQR$ ,  $9A(\Delta PQR) = 16A(\Delta LMN)$  . If  $QR = 20$  then find  $MN$ .

**Ans :**  $\frac{A(\Delta LMN)}{A(\Delta PQR)} = \frac{9}{16} = \frac{MN^2}{QR^2}$  ..... (Ratios of areas of similar triangles)

$$\therefore \frac{MN^2}{QR^2} = \frac{9}{16}$$

$$\therefore \frac{MN}{QR} = \frac{3}{4}$$

$$\therefore \frac{MN}{20} = \frac{3}{4}$$

$$\therefore MN = \frac{3 \times 20}{4}$$

$$\therefore MN = 15 \text{ units}$$

3)  $\triangle ABC$  and  $\triangle DEF$  are equilateral triangles. If  $A(\triangle ABC) : A(\triangle DEF) = 1 : 2$  and  $AB = 4$ , find  $DE$ .

4) If  $\triangle ABC \sim \triangle PQR$ ,  $A(\triangle ABC) = 81 \text{ cm}^2$ ,  $A(\triangle PQR) = 121 \text{ cm}^2$  if  $BC = 6.3 \text{ cm}$ , then find  $QR$ .

5) If  $\triangle ABC \sim \triangle PQR$  and  $AB : PQ = 2 : 3$  Find the value of  $\frac{A(\triangle ABC)}{A(\triangle PQR)}$



6) If  $\Delta ABC \sim \Delta PQR$ ,  $AB : PQ = 4 : 5$  and  $A(\Delta PQR) = 125 \text{ cm}^2$  then find  $A(\Delta ABC)$ .

**Q. 3 A) Activity.**

1) In the given figure X is any point in the interior of the triangle. Point X is joined to the vertices of triangle, Seg PQ  $\parallel$  Seg DE, Seg QR  $\parallel$  Seg EF. Complete the activity and prove that Seg PR  $\parallel$  Seg DF.

**Proof :** In  $\Delta XDE$ , PQ  $\parallel$  DE ..... (Given)

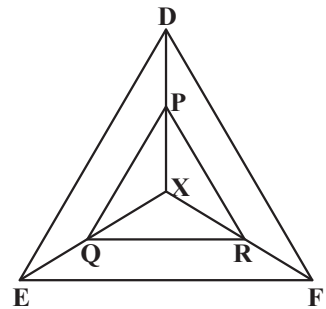
$$\frac{XP}{PD} = \frac{\boxed{\phantom{XQ}}}{QE} \text{ ..... (Basic proportionality theorem) (I)}$$

In  $\Delta XEF$ , QR  $\parallel$  EF ..... (given)

$$\therefore \frac{XQ}{\boxed{\phantom{XQ}}} = \frac{XR}{\boxed{\phantom{XR}}} \text{ ..... (II)}$$

$$\therefore \frac{XP}{PD} = \frac{\boxed{\phantom{XQ}}}{\boxed{\phantom{XR}}} \text{ ..... [From I and II]}$$

seg PR  $\parallel$  seg DE ..... (By converse of basic proportionality theorem)



**Ans : Proof :** In  $\Delta XDE$ , PQ  $\parallel$  DE ..... (Given)

$$\therefore \frac{XP}{PD} = \frac{\boxed{XQ}}{QE} \text{ ..... (Basic proportionality theorem) (I)}$$

In  $\Delta XEF$  QR  $\parallel$  EF ..... (given)

$$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}} \text{ (Basic proportionality theorem) ..... (II)}$$

$$\therefore \frac{XP}{PD} = \frac{XR}{\boxed{RF}} \text{ ..... [from I and II]}$$

seg PR  $\parallel$  seg DF ..... (By converse of basic proportionality theorem)

2) If  $\Delta ABC \sim \Delta PQR$ ,  $A(\Delta ABC) = 81 \text{ cm}^2$ ,  $A(\Delta PQR) = 121 \text{ cm}^2$ ,  $BC = 6.3 \text{ cm}$  then complete the following activity to find QR.

**Activity :**  $\Delta ABC \sim \Delta PQR$  ..... (given)

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\boxed{\phantom{000}}}{QR^2} \dots\dots\dots ( \boxed{\phantom{000}} )$$

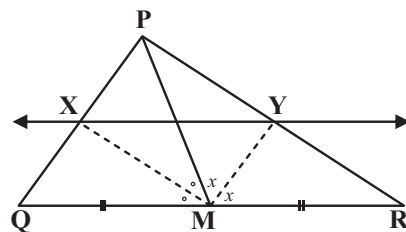
$$\therefore \frac{\boxed{\phantom{000}}}{121} = \frac{(6.3)^2}{QR^2}$$

$$\therefore \frac{\boxed{\phantom{000}}}{11} = \frac{(6.3)}{QR} \dots\dots\dots \text{(Taking square root of both side)}$$

$$\therefore QR = \frac{6.3 \times 11}{\boxed{\phantom{000}}}$$

$$\therefore QR = \boxed{\phantom{000}} \text{ cm}$$

3) In  $\Delta PQR$  seg PM is median. Angle bisectors of  $\angle PMQ$  and  $\angle PMR$  intersect side PQ and side PR in points X and Y respectively. Prove that  $XY \parallel QR$ . Complete the activity by filling in the boxes.



**Activity :** In  $\Delta PMQ$ , ray MX is bisector of  $\angle PMQ$

$$\therefore \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} \dots\dots\dots \text{(I) theorem of angle bisector.}$$

In  $\Delta PMR$  ray MY is bisector of  $\angle PMR$ .

$$\therefore \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} = \frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}}} \dots\dots\dots \text{(II) theorem of angle bisector.}$$

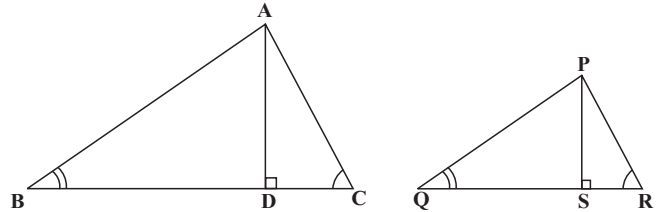
But  $\frac{MP}{MQ} = \frac{MP}{MR}$  ..... M is the midpoint QR, hence  $MQ = MR$

$$\therefore \frac{PX}{XQ} = \frac{PY}{YR}$$

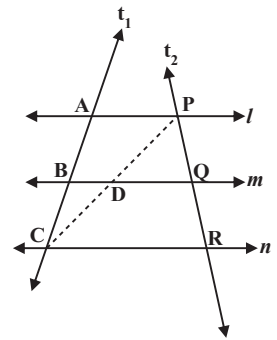
$\therefore XY \parallel QR$  ..... converse of basic proportionality theorem.

**Q. 3 B) Prove the following statements.**

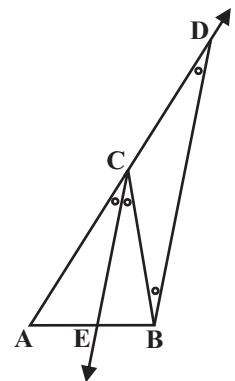
- 1) Prove that "The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines."



- 2) Prove that "When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides."



- 3) Prove that "The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides,"



## 2. Pythagoras Theorem

### Previous Knowledge :

- 1) In  $\Delta ABC$ ,  $m\angle B = 90^\circ$  then  $AC^2 = AB^2 + BC^2$
- 2) In  $\Delta ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = 60^\circ$ ,  $\angle C = 30^\circ$  then  $AB = \frac{1}{2} AC$  and  $BC = \frac{\sqrt{3}}{2} AC$ .
- 3) In  $\Delta ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = \angle C = 45^\circ$  then  $AB = BC = \frac{1}{\sqrt{2}} AC$ .
- 4) In  $\Delta ABC$ ,  $\angle B = 90^\circ$ , seg  $BD \perp$  Hyp  $AC$  then  $BD^2 = AD \times DC$ .
- 5) In  $\Delta ABC$  If  $D$  is the midpoint of side  $BC$  then  $AB^2 + AC^2 = 2AD^2 + 2BD^2$

### Q. 1 A) Choose the correct alternative.

- 1) In a right - angled triangle, if sum of the squares of the sides making a right angle is 169, then what is the length of the hypotenuse ?  
(A) 15                      (B) 13                      (C) 5                      (D) 12

**Ans :** (B)

Sum of squares of sides = hypotenuse square.

$\therefore (13)^2 = 169$	hypotenuse square
$\therefore \text{hypotenuse} = 13$	

- 2) Out of the following which is the Pythagorean triplet ?  
(A) 1, 5, 10                      (B) 3, 4, 5  
(C) 2, 2, 2                      (D) 5, 5, 2

**Ans :**  $3^2 + 4^2 = 5^2$

- 3) Out of the dates given below which date constitutes a Pythagorean triplet.  
(A) 15/08/17                      (B) 16/08/16  
(C) 03/05/17                      (D) 04/09/15

**Ans :**

- 4) If  $a, b, c$  are sides of triangle and  $a^2 + b^2 = c^2$  name the type of triangle.  
(A) Obtuse angled triangle                      (B) Acute angled triangle  
(C) Right angled triangle                      (D) Equilateral triangle

**Ans :**

5) Find perimeter of a square if its diagonal is  $10\sqrt{2}$  cm

(A) 10 cm

(B)  $40\sqrt{2}$  cm

(C) 20 cm

(D) 40 cm

**Ans :**

6) Altitude on the hypotenuse of a right-angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

(A) 9 cm

(B) 4 cm

(C) 6 cm

(D) 2.6 cm

**Ans :**

7) Height and base of a right - angled triangle are 24 cm and 18 cm find the length of its hypotenuse.

(A) 24 cm

(B) 30 cm

(C) 15 cm

(D) 18 cm

**Ans :**

**Q. 1 B) Attempt the following.**

1) In  $\Delta RST$ ,  $\angle S = 90^\circ$ ,  $\angle T = 30^\circ$ ,  $RT = 12$  cm then find  $RS$  and  $ST$ .

$$\text{Ans : } RS = \frac{1}{2} RT = 6 \text{ cm}$$

$$ST = \frac{\sqrt{3}}{2} RT = 6\sqrt{3} \text{ cm}$$

2) Find the side of the square whose diagonal is  $10\sqrt{2}$ .

$$\text{Ans : } \text{side of the square} = \frac{1}{\sqrt{2}} \times \text{diagonal}$$

$$= \frac{1}{\sqrt{2}} \times 10\sqrt{2} = 10 \text{ unit}$$

3) In  $\Delta ABC$ , if  $AB^2 = AC^2 + CB^2$ , state with reason whether  $\Delta ABC$  is right angled triangle or not.

**Ans :**

4) Do the sides 7 cm 24 cm and 25 cm form a right - angled triangle? Give reason.

**Ans :**

5) Find the length a diagonal of a rectangle having side 11 cm and 60 cm.

**Ans :**

6) Find the length of the hypotenuse of a right - angled triangle if remaining side are 9 cm and 12 cm

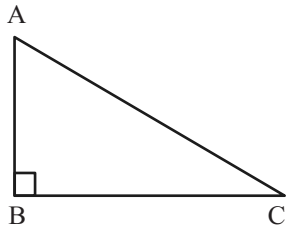
**Ans :**

7) Find the length of the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

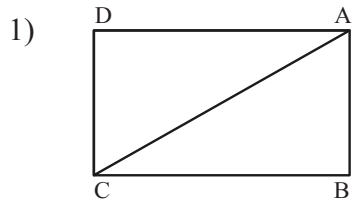
**Ans :**

8) In  $\triangle ABC$ ,  $\angle ABC = 90^\circ$ ,  $\angle BAC = \angle BCA = 45^\circ$ . If  $AC = 9\sqrt{2}$  then find the value of AB.

Ans :



Q. 2 A) Complete and write the following activities.



In the above figure  $\square ABCD$  is rectangle.

If  $AB = 5$ ,  $AC = 13$  then complete the following activity to find BC.

Activity :  $\triangle ABC$  is  triangle

$\therefore$  By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\therefore 25 + BC^2 = \text{$$

$$\therefore BC^2 = \text{$$

$$\therefore BC = \text{$$

Ans :  $\triangle ABC$  is  triangle

$\therefore$  By Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\therefore 25 + BC^2 = \text{$$

$$\therefore BC^2 = \text{$$

$$\therefore BC = \text{$$

2) In the given figure,

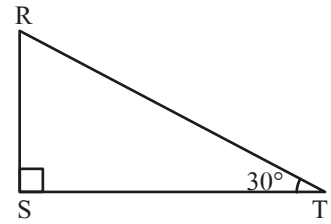
**Activity :**  $\Delta RST$  is a  triangle.

$$RS = \text{} \times RT$$

$$RS = 6 \text{ cm}$$

$$ST = \text{} \times RT$$

$$ST = \text{} \text{ cm.}$$



**Ans :** In the given figure,

$\Delta RST$    $30^\circ - 60^\circ - 90^\circ$  triangle.

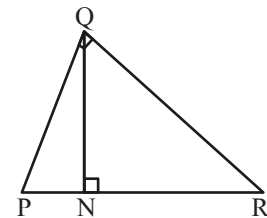
$$\therefore RS = \text{} \times RT$$

$$\therefore RS = 6 \text{ cm}$$

$$\therefore ST = \text{} \times RT$$

$$\therefore ST = \text{} \text{ cm}$$

3) In the given figure,  $\angle PQR = 90^\circ$ ,  
 seg  $QN \perp$  seg  $PR$ ,  $PN = 9$ ,  $QN = 12$   
 Find  $NR$ . Fill in the boxes to find  $NR$ .



**Activity :** In  $\Delta PQR$ ,

$$\angle PQR = 90^\circ$$

seg  $QN \perp$  seg  $PR$

$$\therefore NQ^2 = \text{} \dots\dots\dots \text{[Theorem of Geometric mean of a triangle]}$$

$$\therefore 12^2 = \text{} \dots\dots\dots \text{[Substituting the given values]}$$

$$\therefore NR = \frac{\text{}}{9}$$

$$\therefore NR = \text{}$$



**Q. 2 B) Attempt the following.**

- 1) The length of hypotenuse of a right - angled triangle is 15 cm. Find the length of median of its hypotenuse.

$$\begin{aligned}\text{Ans : } \quad \text{median} &= \frac{1}{2} \text{ hypotenuse} \\ &= \frac{1}{2} \times 15 \\ &= 7.5 \text{ cm}\end{aligned}$$

- 2) Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq. cm.

$$\text{Ans : } \text{Area of rectangle} = \text{length} \times \text{breadth}$$

$$\therefore \text{breadth} = \frac{\text{area of rectangle}}{\text{length}}$$

$$\therefore = \frac{192}{16}$$

$$\therefore = 12 \text{ cm}$$

now by pythagoras theorem

$$\text{diagonal}^2 = \text{length}^2 + \text{breadth}^2$$

$$= 16^2 + 12^2$$

$$= 256 + 144$$

$$= 400$$

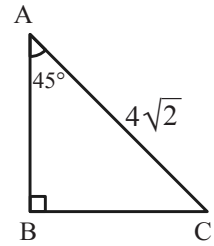
$$= \sqrt{400} = 20 \text{ cm}$$

- 3) Identify, with reason, whether (24, 70, 74) is a pythagorean triplet or not.

**Ans :**

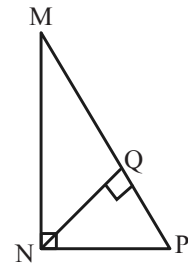
4) In  $\triangle ABC$ ,  $\angle ABC = 90^\circ$ ,  $\angle BAC = 45^\circ$  and  $AC = 4\sqrt{2}$  cm find AB.

**Ans :**



5) In the figure,  $\angle MNP = 90^\circ$ , seg  $NQ \perp$  seg MP,  $MQ = 9$ ,  $QP = 4$ , find NQ.

**Ans :**



6) In  $\triangle PQR$ ,  $\angle P = 60^\circ$ ,  $\angle Q = 90^\circ$ , and  $QR = 6.3$  cm then find the values of PR and PQ.

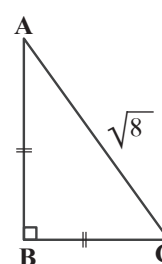
7) In  $\triangle ABC$ , seg AP is a median. If  $BC = 18$ ,  $AB^2 + AC^2 = 260$ . Find AP.

- 8) In  $\triangle ABC$ ,  $AB = 9$  cm  $BC = 40$  cm and  $AC = 41$  cm. State whether  $\triangle ABC$  is a right-angled triangle or not ? Write reason.

**Q. 3 A) Activity**

1) Complete the activity.

$$\begin{aligned}
 AB = BC & \dots\dots\dots \boxed{\phantom{0000}} \\
 \therefore \angle BAC & = \boxed{\phantom{0000}} \\
 \therefore AB = BC & = \boxed{\phantom{0000}} \times AC \\
 & = \boxed{\phantom{0000}} \times \sqrt{8} \\
 & = \boxed{\phantom{0000}} \times 2\sqrt{2} \\
 & = \boxed{\phantom{0000}}
 \end{aligned}$$

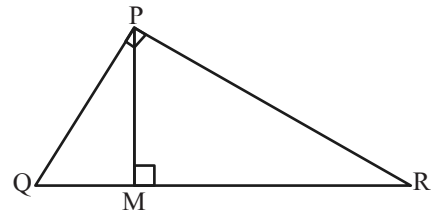


**Ans :**  $AB = BC$  ..... Given

$$\therefore \angle BAC = \boxed{\angle BCA / 45^\circ}$$

$$\begin{aligned}
 \therefore AB = BC & = \boxed{\frac{1}{\sqrt{2}}} \times AC \\
 & = \boxed{\frac{1}{\sqrt{2}}} \times \sqrt{8} \\
 & = \boxed{\frac{1}{\sqrt{2}}} \times 2\sqrt{2} \\
 & = \boxed{2}
 \end{aligned}$$

2) In the given figure  $\angle QPR = 90^\circ$  seg  $PM \perp$  seg  $QR$  and  $Q-M-R$ ,  $PM = 10$ ,  $QM = 8$ , then complete the following activity to find the value of  $QR$ .



**Activity :**  $\Delta PQR$ ,  $\angle QPR = 90^\circ$

and seg  $PM \perp$  seg  $QR$

$$\therefore PM^2 = \boxed{\phantom{000}} \times MR \dots\dots\dots \boxed{\phantom{000}}$$

$$\therefore (\boxed{\phantom{000}})^2 = 8 \times MR$$

$$\therefore \frac{100}{8} = MR$$

$$\therefore \boxed{\phantom{000}} = MR$$

Now,  $QR = QM + MR \dots\dots\dots (\because Q-M-R)$

$$\therefore QR = 8 + \boxed{\phantom{000}}$$

$$\therefore QR = \boxed{\phantom{000}}$$

**Ans :** In  $\Delta PQR$ ,  $\angle QPR = 90^\circ$  and seg  $PM \perp$  seg  $QR$ .

$$\therefore PM^2 = \boxed{QM} \times MR \dots\dots\dots \text{Theorem of Geometric mean}$$

$$\therefore (\boxed{10})^2 = 8 \times MR$$

$$\therefore \frac{100}{8} = MR$$

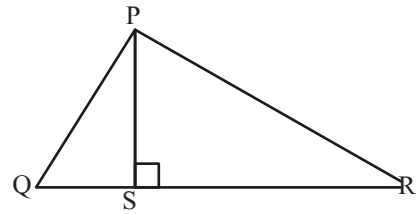
$$\therefore \boxed{12.5} = MR$$

Now,  $QR = QM + MR \dots\dots\dots (\because Q-M-R)$

$$\therefore QR = 8 + \boxed{12.5}$$

$$\therefore QR = \boxed{20.5}$$

- 3) In  $\Delta PQR$ , seg  $PS \perp$  side  $QR$ , then complete the activity to prove  $PQ^2 + RS^2 = PR^2 + QS^2$



**Activity :** In  $\Delta PSQ$ ,  $\angle PSQ = 90^\circ$

$$PS^2 + QS^2 = PQ^2 \dots\dots\dots \text{(Pythagoras theorem)}$$

$$PS^2 = PQ^2 - \boxed{\phantom{000}} \dots\dots\dots \text{(I)}$$

Similarly, In  $\Delta PSR$ ,  $\angle PSR = 90^\circ$

$$\therefore PS^2 + \boxed{\phantom{000}} = PR^2 \dots\dots\dots \text{(Pythagoras theorem)}$$

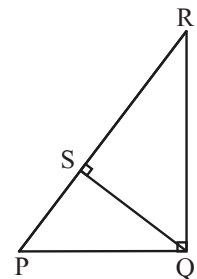
$$\therefore PS^2 = PR^2 - \boxed{\phantom{000}} \dots\dots\dots \text{(II)}$$

$$\therefore PQ^2 - \boxed{\phantom{000}} = \boxed{\phantom{000}} - RS^2 \dots\dots \text{from (I) and (II)}$$

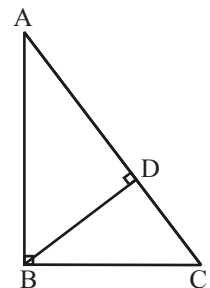
$$\therefore PQ^2 + \boxed{\phantom{000}} = PR^2 + QS^2$$

**Q. 3 B) Prove the following.**

- 1) Prove that : 'In a right - angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.'



- 2) Prove that : "In a right - angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two side."



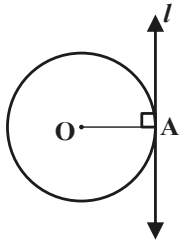
### 3. Circle

**Previous Knowledge :**

- **Circle :** Radius, diameter, chords, tangents of the circle, secant, central angle etc.

**Concepts :**

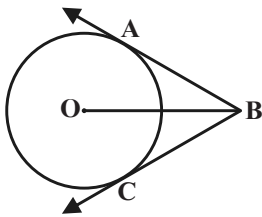
**1) Tangent theorem**



Line  $l$  is a tangent to the circle with centre  $O$  at the point 'A' then line  $l \perp$  radius  $OA$ .

**2) Tangent segment theorem :**

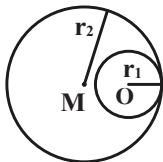
Tangent segments drawn from an external point to a circle are congruent.



$$BA \cong BC$$

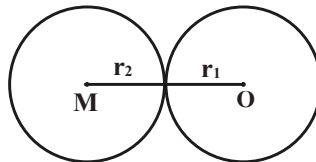
**3) Touching circles :**

Internally touching circles



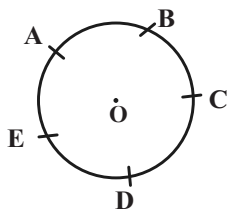
$$d(M, O) = r_2 - r_1$$

Externally touching circles



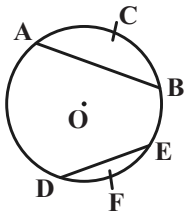
$$d(M, O) = r_1 + r_2$$

**4) Property of sum of measures of areas :**



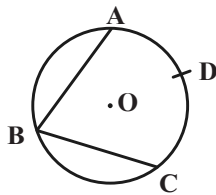
$$m(\text{arc } ABC) + m(\text{arc } CD) = m(\text{arc } ABD)$$

- 5) The chords corresponding to congruent arcs of a circle (or congruent circles) are congruent.



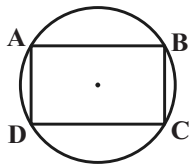
If arc  $ACB \cong$  arc  $DFE$  then chord  $AB \cong$  chord  $DE$

- 6) Inscribed angle



$$\angle ABC = \frac{1}{2} m(\text{arc } ADC)$$

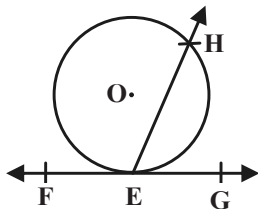
- 7) Cyclic Quadrilateral



1)  $\angle A + \angle C = 180^\circ$

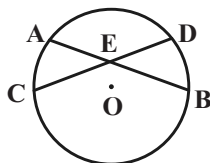
2)  $\angle B + \angle D = 180^\circ$

- 8) Theorem of angle between tangent and secant.



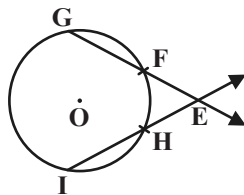
$$\angle HEG = \frac{1}{2} m(\text{arc } HE)$$

- 9) Theorem of internal division of chords.



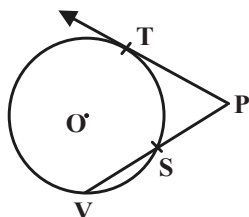
$$EA \times EB = ED \times EC$$

- 10) Theorem of external division of chords.



$$EF \times EG = EH \times EI$$

11) Tangent secant segments theorem.



$$PT^2 = PS \times PV$$

**Examples :**

**Q. 1 A) Select correct alternative.**

- 1) Two circles of radii 5.7 cm and 3.3 cm touch each other externally. Find the distance between their centres.

(A) 2.4 cm      (B) 1.2 cm      (C) 9.0 cm      (D) 4.5 cm

Let radii of circles are  $r_1$  and  $r_2$

$$r_1 = 5.7 \text{ cm}, \quad r_2 = 3.3 \text{ cm}$$

Externally touching circles

$\therefore$  Distance between their centres

$$= r_1 + r_2$$

$$= 5.7 + 3.3 = 9.0$$

**Ans :** Correct alternative (C)

- 2) How many circles can be drawn passing through three non collinear points?

(A) 0      (B) infinite      (C) 2      (D) one and only one

**Ans :** Correct alternative (D)

- 3) Angles inscribed in the same arc are .....

(A) unequal      (B) right angled      (C) congruent      (D) acute angled

**Ans :** Correct alternative (C)

- 4)  $\angle PQR$  is inscribed in arc PQR of a circle with centre 'O'. If  $m\angle PQR 100^\circ$ . Find  $m(\text{arc PQR})$ ?

(A)  $100^\circ$       (B)  $80^\circ$       (C)  $50^\circ$       (D)  $260^\circ$

**Ans :** Correct alternative (.....)

- 5) Length of the biggest chord of a circle is 16 cm, find its radius?

(A) 8 cm      (B) 16 cm      (C) 32 cm      (D) 8.5 cm

**Ans :** Correct alternative (.....)



**Q. 1 B) Solve.**

1) □ ABCD is a cyclic quadrilateral. If  $\angle D = 105^\circ$ , find the  $m\angle B$ ?

$$\angle B + \angle D = 180^\circ \text{ (Theorem of cyclic)}$$

$$\therefore \angle B + 105^\circ = 180^\circ$$

$$\therefore \angle B = 180^\circ - 105^\circ$$

$$\boxed{\angle B = 75^\circ}$$

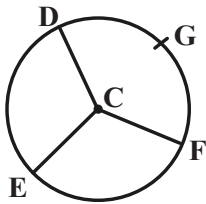
2) What is the measure of half circle?

**Ans :**  $180^\circ$

3) What is the distance between two parallel tangents of a circle having radius 5.4 cm?

**Ans :**

4) Points G, D, E, F are concyclic points of a circle with centre C.  $m$  (arc DGF)  $210^\circ$ , find  $m$  (arc DEF)?



**Q. 2 A) Complete the activity.**

- 1) In fig. seg PS is a tangent segment of a circle. Line PR is secant.

If  $PQ = 3.6$ ,  $PR = 10$  then find PS.

$$PS^2 = PQ \times \boxed{\phantom{000}} \text{ Tangent}$$

secant segments theorem

$$= 3.6 \times \boxed{\phantom{000}}$$

$$= \boxed{\phantom{000}}$$

$$PS = \boxed{\phantom{000}}$$

**Ans.**

If  $PQ = 3.6$ ,  
 $PR = 10$  then find PS.

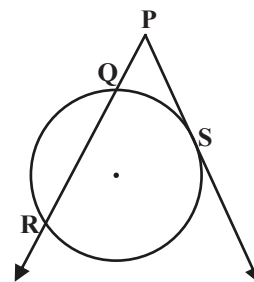
$$PS^2 = PQ \times \boxed{PR} \text{ Tangent}$$

secant segments theorem

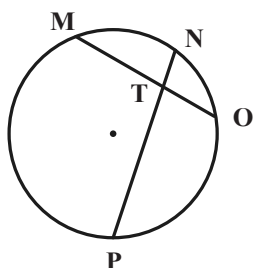
$$= 3.6 \times \boxed{10}$$

$$= \boxed{36}$$

$$PS = \boxed{6}$$



- 2) Chord MO and chord PN are intersecting in point 'T'.



If  $MT = 6$ ,  $TO = 3$ ,  $NT = 2.5$ , find TP.

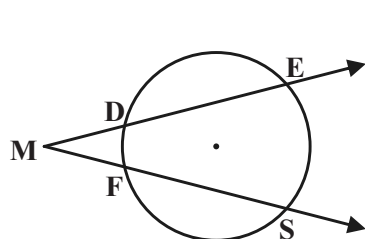
$$TM \times TO = \boxed{\phantom{000}} \times TP$$

$$6 \times 3 = 2.5 \times \boxed{\phantom{000}}$$

$$\boxed{\phantom{000}} = 2.5 \times TP$$

$$\boxed{\phantom{000}} = TP$$

- 3) If  $m(\text{arc ES}) = 110^\circ$ ,  $m(\text{arc DF}) = 52^\circ$  then find  $m\angle EMS = ?$



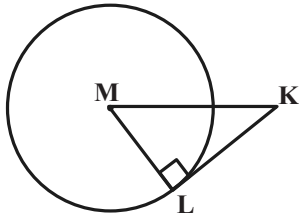
$$\angle EMS = \frac{1}{2} m(\text{arc } \boxed{\phantom{000}} - m \text{ arc } \boxed{\phantom{000}})$$

$$= \frac{1}{2} (110^\circ - \boxed{\phantom{000}})$$

$$\angle EMS = \boxed{\phantom{000}}$$

**Q. 2 B) Solve.**

1) KL is tangent segment of a circle with centre M. If MK=12, ML=6, find LK ?



**Ans. :**

If  $\triangle MLK$   $\angle L = 90^\circ$

Tangent theorem

$$MK^2 = ML^2 + LK^2$$

Pythagoras theorem

$$(12)^2 = (6)^2 + LK^2$$

$$\therefore 144 - 36 = LK^2$$

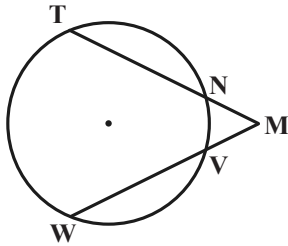
$$\therefore 108 = LK^2$$

$$\therefore \sqrt{12 \times 9} = LK$$

$$\therefore LK = 6\sqrt{3}$$

2) Chord TN and chord WV are intersecting each other in point M.

MV = 6, VW = 10, MN = 8, find MT = ?



**Q. 3 A) Complete the activity.**

1) Line BC is a tangent of a circle with centre M.

$$\angle ABE = \frac{1}{2} m(\text{arc AFB})$$

$$\angle ABE + \angle ABC = \boxed{\phantom{000}}$$

Linear pair of angles

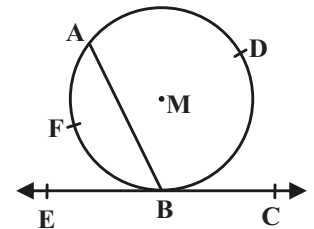
$$\frac{1}{2} m(\text{arc AFB}) = 180 - \boxed{\phantom{000}}$$

$$\angle ABC = 180 - \frac{1}{2} m(\text{arc AFB})$$

$$= 180 - \frac{1}{2} m(360 - \boxed{\phantom{000}})$$

$$= 180 - 180 + \frac{1}{2} \boxed{\phantom{000}}$$

$$\angle ABC = \frac{1}{2} m(\text{arc } \boxed{\phantom{000}})$$



$$\angle ABE = \frac{1}{2} m(\text{arc AFB})$$

$$\angle ABE + \angle ABC = \boxed{180^\circ}$$

Linear pair of angles

$$\frac{1}{2} m(\text{arc AFB}) = 180 - \boxed{\angle ABC}$$

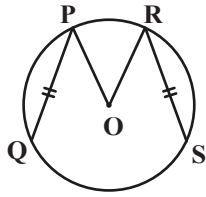
$$\angle ABC = 180 - \frac{1}{2} m(\text{arc AFB})$$

$$= 180 - \frac{1}{2} m(360 - \boxed{\text{arc ADB}})$$

$$= 180 - 180 + \frac{1}{2} \boxed{m(\text{कंस ADB})}$$

$$\angle ABC = \frac{1}{2} m(\text{arc } \boxed{\text{ADB}})$$

2) In fig. chord  $PQ \cong$  chord  $RS$ ,  $\angle POR = 100^\circ$ ,  $m(\text{arc } PQ) = 70^\circ$  then,



$$m(\text{arc } PR) = \boxed{\phantom{000}} \text{ Central angle}$$

$$m(\text{arc } PR) =$$

$$m(\text{arc } PQ) = m(\text{arc } \boxed{\phantom{000}}) \text{ arcs made by congruent chords.}$$

$$m(\text{arc } PQ) + m(\text{arc } QS) + m(\text{arc } \boxed{\phantom{000}}) + m(\text{arc } \boxed{\phantom{000}}) = 360^\circ$$

$$m(\text{arc } QS) = \boxed{\phantom{000}}$$

**Q. 3 B) Solve.**

1) Prove that 'An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.'

**Given** :  $\square ABCD$  is a cyclic quadrilateral.

$\angle CDE$  is an exterior angle of  $\square ABCD$ .

**To prove** :  $\angle CDE \cong \angle CBA$

**Proof** : 1)  $\square ABCD$  is cyclic quadrilateral.

$$\angle ABC + \angle CDA = 180^\circ \text{ (cyclic quadrilateral theorem)}$$

$$2) \angle CDE + \angle CDA = 180^\circ \text{ (Linear pair of angles)}$$

$$3) \angle ABC + \angle CDA = \angle CDE + \angle CDA \text{ ..... from (1), (2)}$$

$$4) \therefore \angle ABC = \angle CDE$$

$$\therefore \angle CDE \cong \angle CBA$$

2) Prove.

Tangent segments drawn from an external point to a circle are congruent.

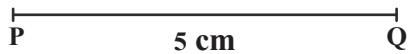
## 4. Geometric Construction

Previous Knowledge :

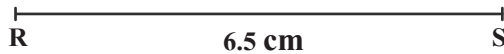
**Q. 1 A) To draw a line segment of given length.**

- 1) Draw line segment PQ of length 5 cm.

While drawing the line segment PQ using ruler, start measuring from the zero (0) mark on the ruler.



- 2)  $l(RS) = 6.5$  draw RS.



### Question for practice

- i) Draw line segment of AB of length 6 cm.

- ii) If  $l(MN) = 4.5$  cm then draw MN.

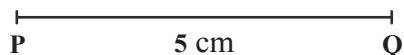
- iii) Draw line segment CD such that  $l(CD) = 7.2$  cm.

**B) Perpendicular bisector of a line segment.**

- 1) Draw a line segment PQ of length 5 cm and draw its perpendicular bisector.

**Steps :**

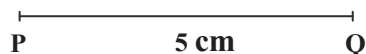
- i) Draw a line segment PQ of length 5 cm using ruler.



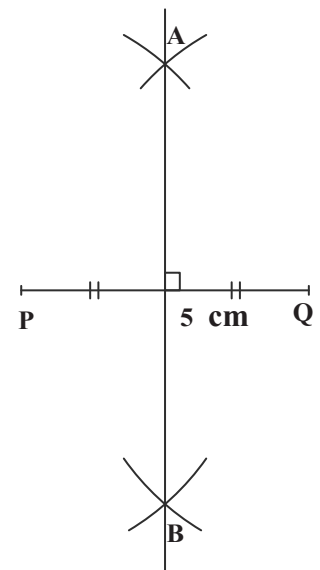
- ii) To draw perpendicular bisector using compass adjust the compass with a length of a little more than half of the length of PQ. Place the compass pointer at point P and draw arcs above and below the line segment.



- iii) Keeping the same length in the compass, place the compass pointer at point Q. Draw arcs as mentioned in step ii) intersecting the previously drawn arc. Name the points of intersecting as point A and B.



- iv) Using ruler, join points A and B. This line is perpendicular bisector of line segment PQ.



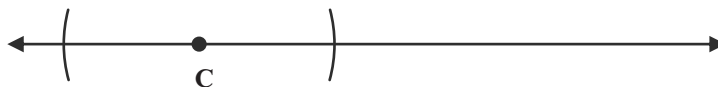
**C) Construction of perpendicular to a line through a point on it.**

Draw a line. Mark any point C on it. To draw perpendicular line through the point C.

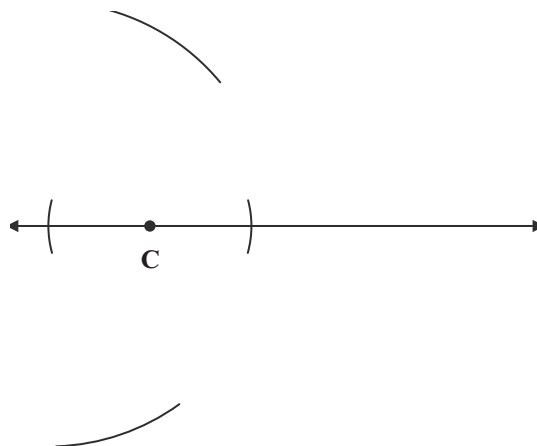
- i) Draw a line and mark any point C on it.



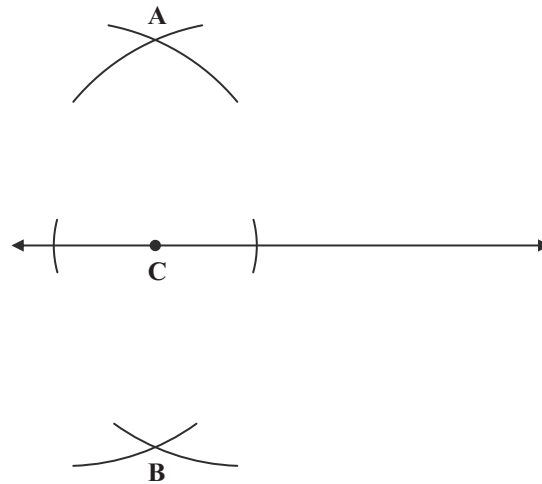
- ii) Place a compass pointer at point C. Take suitable distance between two arms of the compass and draw arcs, intersecting the line on both the sides of point C.



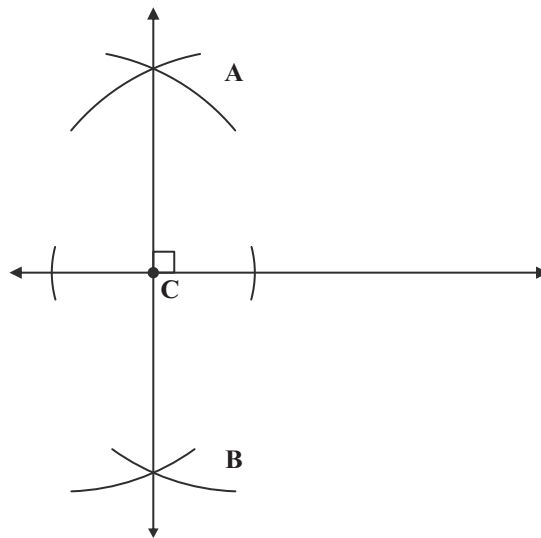
- iii) Now put the compass pointer at point of intersection, to the left of point C and by taking suitable distance between two arms of the compass, mark arcs above and below the given line.



- iv) Keeping the same length in the compass, place the compass pointer at point of intersection to the right of point C and draw intersecting arcs to previously drawn arcs. Name the points of intersection as point A and B.

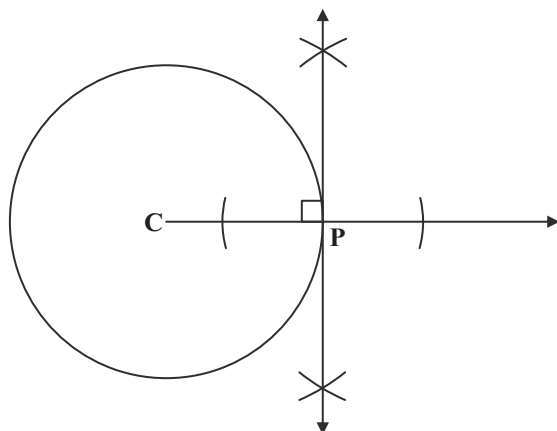


- v) Draw a line joining points A and B. Line AB is perpendicular to a given line at point C.





**D) Construction of a tangent to a circle at a point on the circle.**



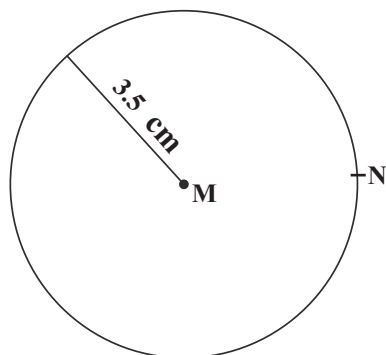
To construct a tangent passing through a point P on the circle with centre C.

**Important :** For this we will use the construction of drawing a perpendicular to a line through a point on it.

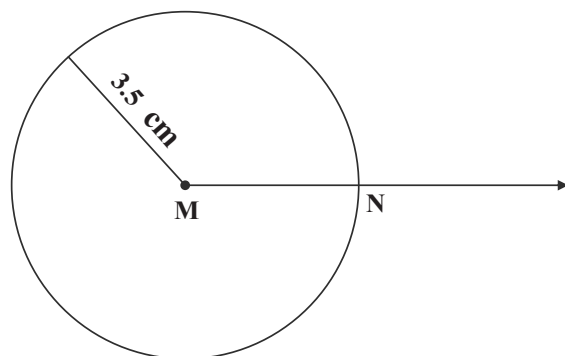
**Example :** Construct tangent to a circle with centre M and radius 3.5 cm. at any point N on it.

**Steps of Construction :**

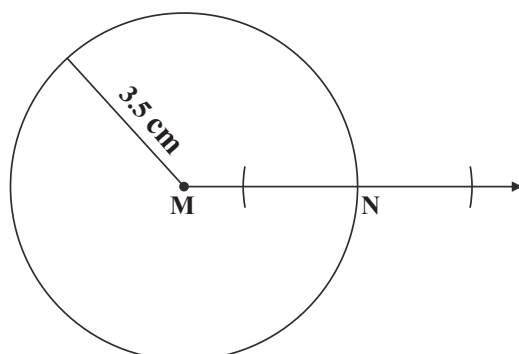
i) Draw a circle with center M and radius 3.5 cm. Take any point N on the circle.



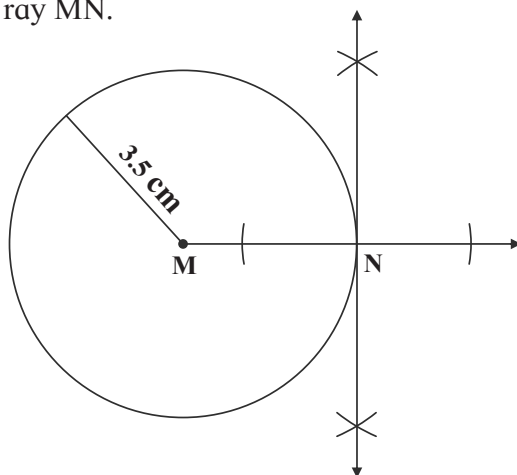
ii) Draw ray MN.



- iii) Draw a perpendicular line to ray MN through point N. This perpendicular line is required tangent to the circle at point N.



- iv) Place the compass pointer on point N. Draw arcs on both the sides of point N intersecting ray MN.



- v) Using points of intersection of arc and ray MN draw intersecting arcs above and below ray MN. Draw a line through these points of intersection of arcs. This line is required tangent to a circle at point N.

### Question for practice

- 1) Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it.
- 2) Construct a tangent to a circle with centre P and radius 3.5 cm at any point M on it. (Aug. 2022)
- 3) Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at point P and Q ( March 2023)

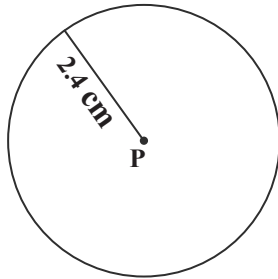
(Hint : Draw a circle of radius 3.3 cm. Chord of length 6.6 cm is the diameter of a circle. ( $2 \times \text{radius} = \text{diameter}$ ) Construct tangents through points P and Q diameter PQ.

**E) To construct tangents to a circle from a point outside the circle.**

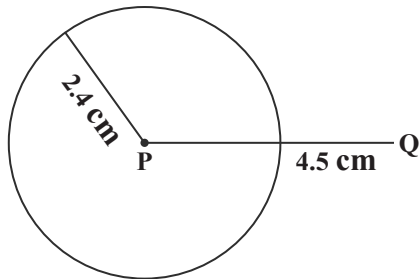
**Example :** Draw a circle with centre P and radius 2.4 cm. Take point Q at a distance 4.5 cm from the centre. Construct tangents to the circle from point Q.

**Steps of Construction :**

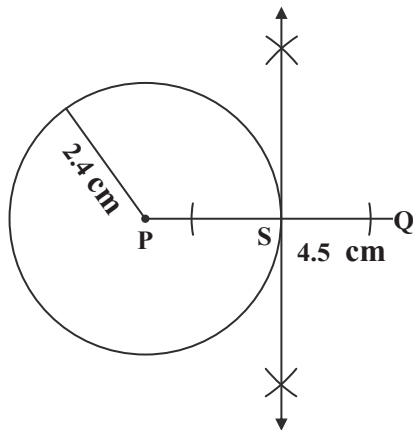
- i) Draw a circle with centre P and radius 2.4 cm.



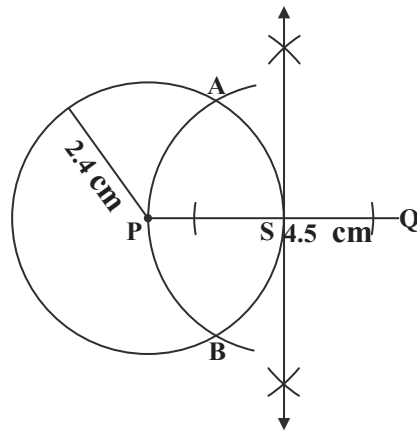
- ii) Take a point Q at 4.5 cm from centre P. Draw segment PQ.



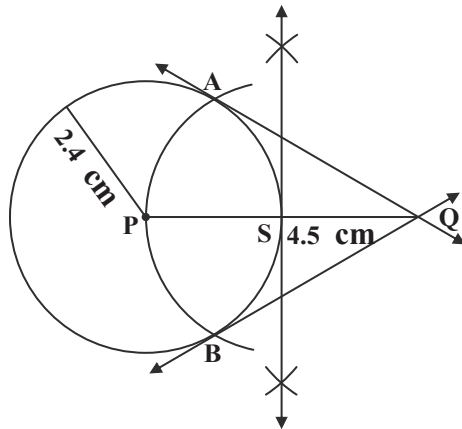
- iii) Draw perpendicular bisector of seg PQ. Name the midpoint of seg PQ as 'S'.



- iv) Draw an arc with radius  $SP$  and centre  $S$ . This arc will intersect circle at two distinct points. Name these points as  $A$  and  $B$ .



- v) Draw line  $QA$  and  $QB$ . These two lines are required tangents.



### Question for practice

- 1) Draw a circle with centre  $O$  and radius  $3.5$  cm. take point  $P$  at distance  $7.5$  cm from the centre. Construct tangents to the circle from point  $P$  (Aug. - 2022).
- 2) Draw a circle with centre  $P$  and radius  $3.4$  cm . Take point  $Q$  distance  $5.5$  cm from the centre. Construct tangents to the circle from point  $Q$ .
- 3) Draw a circle with radius  $4.1$  cm. Construct tangents to the circle from a point at a distance  $7.3$  cm from the centre.

## Construction of triangle

Previous Knowledge :

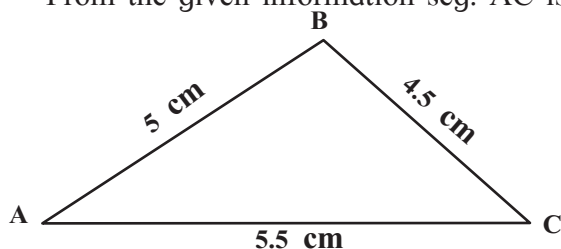
**Q. 1) S-S-S Construction :** Construction of a triangle when length of three sides are given.

**Example :**  $\triangle ABC$   $l(AB) = 5 \text{ cm}$  ,  $l(BC) = 4.5 \text{ cm}$  ,  $l(AC) = 5.5 \text{ cm}$

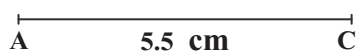
**Steps of Construction :**

i) Draw analytical figure.

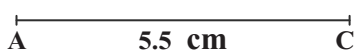
From the given information seg. AC is longest. So seg. AC is a base of  $\triangle ABC$



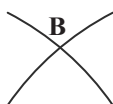
ii) Using ruler draw seg AC of 5.5 cm.



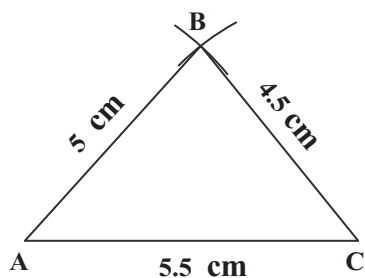
iii)  $l(AB) = 5 \text{ cm}$  take a distance of 5 cm in both the arms of compass and keeping pointer on point A draw an arc above seg. AC



iv)  $l(BC) = 4.5 \text{ cm}$ . For making point B, take a distance of 4.5 cm between two arms of a compass and by keeping pointer on point C draw an arc intersecting the previous arc. Point of intersecting of these two arcs is point B.



v) Join points A and B join points B and C. This is required  $\triangle ABC$ .




---

**Question for practice**

---

- 1) Draw  $\triangle PQR$  such that  $l(PQ) = 4$  cm,  $l(QR) = 5$  cm  $l(PR) = 3$  cm.
  - 2) In  $\triangle MNT$   $l(MN) = l(MT) = 5$  cm,  $l(NT) = 6$  cm. Draw  $\triangle MNT$
  - 3) Construct  $\triangle ABC$  such that  $l(AC) = 6$  cm,  $l(AB) = 5.4$  cm,  $l(BC) = 4.2$  cm
- 

**Questions of 1 mark.**

Select the correct alternative.

**(1 mark for each question)**

- 1) The number of tangents that can be drawn to a circle at a point on the circle is .....  
 (A) 3                                      (B) 2                                      (C) 1                                      (D) 0
- 2) The maximum number of tangents that can be drawn to a circle from a point outside it is .....  
 (A) 2                                      (B) 1  
 (C) One and only one                      (D) 0
- 3) If  $\triangle ABC \sim \triangle PQR$  and  $\frac{AB}{PQ} = \frac{7}{5}$ , then .....  
 (A)  $\triangle ABC$  is bigger  
 (B)  $\triangle PQR$  is bigger  
 (C) Both triangles will be equal  
 (D) Cannot be decided.

## 5. Co-ordinate Geometry

### 1) Distance Formula :

- 1) If A  $(x_1, y_1)$  and B  $(x_2, y_2)$  be any two points in a plane then the distance between the segment joining point A and B is given by,

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### 2) Section Formula :

If A  $(x_1, y_1)$  and B  $(x_2, y_2)$  be any two points in a plane and point P  $(x, y)$  lies on segment AB such that P divides segAB in the ratio  $m : n$  then Co-ordinates of P is given by

section formula  $x = \frac{mx_2 + nx_1}{m + n}$  and  $y = \frac{my_2 + ny_1}{m + n}$

### 3) midpoint formula :

If A  $(x_1, y_1)$  and B  $(x_2, y_2)$  be any two points in a plane and point P  $(x, y)$  lies on segment AB such that P is midpoint of segAB Co-ordinates of P is given by midpoint formula

$$x = \frac{x_2 + x_1}{2} \quad \text{and} \quad y = \frac{y_2 + y_1}{2}$$

### 4) Centroid Formula :

If A  $(x_1, y_1)$ , B  $(x_2, y_2)$  and C  $(x_3, y_3)$  be vertices of triangle ABC and point G  $(x, y)$  is

Given by centroid formula  $x = \frac{x_1 + x_2 + x_3}{3}$  and  $y = \frac{y_1 + y_2 + y_3}{3}$

### 5) Slope of y axis is not define.

- 6) If A  $(x_1, y_1)$  and B  $(x_2, y_2)$  be any two points in a plane then slope of AB is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- 7) If inclination of line (angle with positive X axis) is  $\theta^\circ$  then slope of line is  $m = \tan\theta^\circ$

- 8) If two lines are parallel then their slopes are equal  $m_1 = m_2$

- 9) If two lines are perpendicular then product of their slopes is -1 that is  $m_1 m_2 = -1$

- 10) If points A, B and C are collinear then slope of AB = slope of BC

**Q. 1 A) - MCQ**

1) Seg PQ is parallel to Y axis and co-ordinates of point P(2, 5) then the co-ordinates of point Q is

- A) (3, 4)                      B) (2, 8)                      C) (5, 5)                      D) (3, 6)

**Explanation :** seg PQ is parallel to  $y$  axis hence its  $x$  number is same therefore  
**Ans.** B (2, 5)

2) The distance between A(3, 4) and origin is

- A) 25                              B) 7                              C) 13                              D) 5

**Explanation :** By distance formula  $d(A,B)\sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{(25)^2} = 5$   
**Ans.** D (5)

---

**Questions for practice**

---

1) Seg LM is parallel to X axis and co-ordinates of point L (-4,5) then the co-ordinates of point M is

- A) (3, 4)                      B) (2, 7)                      C) (5, 5)                      D) (3, 6)

2) The distance between the pair of points A(3, 4) and B(4, 5)

- A)  $\sqrt{5}$       B)  $\sqrt{2}$       C)  $\sqrt{7}$       B)  $\sqrt{5}$

---

**Q.1 B) Attempt the following. (mark for each question)**

1) Find the slope of line which makes angle with positive direction of X axis is  $45^\circ$

**Ans.** If inclination of line (angle with positive X axis) is  $\theta^\circ$  then slope of line is  
 $m = \tan\theta^\circ$  slope of line is  $m = \tan 45^\circ = 1$

2) Find the slope of line passing through A(2, 3) and B(4, 7)

**Ans.** A(2, 3) and B(4, 7) slope  $m = \frac{y_2 - y_1}{x_2 - x_1}$  ,  $m = \frac{7 - 3}{4 - 2} = \frac{4}{2} = 2$

---

**Questions for practice**

---

1) Find the slope of line which makes angle with positive direction of X axis is  $30^\circ$

2) Find the slope of line passing through P(0, 3) and Q(4, 5)



**Q. 2 A Complete the following activity. 2 Marks for each question**

1) To find the midpoint of the segment joining the point A(3, 7) and B(5, 3)

Let the coordinates of the midpoint of segment be (x, y)

**Using midpoint formula :**

$$x = \frac{[3] + 5}{2} \text{ and } y = \frac{7 + [3]}{2}$$

$$x = \frac{8}{2} \text{ and } y = \frac{10}{2}$$

$$x = [4] \quad y = [5]$$

2) To find the value of k if B(k,-5) and C(1,2) and slope of line is 7.

$$\text{Slope of line } m = \frac{y_2 - y_1}{x_2 - x_1}, \quad \frac{2 - [-5]}{1 - k} = 7$$

$$[ 1 ] = 1 - k$$

$$k = [ 1 ] - 1$$

$$k = [ 0 ]$$

**Questions for practice**

1) Find value of k if P(2, 4), Q(3, 6), R(3, 1), S(5, K) and PQ || RS

Slope of PQ = Slope of RS

$$\frac{6 - 4}{3 - [ ]} = \frac{k - [ ]}{5 - 3}$$

$$4 = k - [ ]$$

$$k = [ ]$$

2) Find the co-ordinates of centroid of triangle ABC whose vertices are A(1, 4), B(3, 5), and C(2, 6)

$$\text{centroid formula } x = \frac{x_1 + x_2 + x_3}{3} \text{ and } y = \frac{y_1 + y_2 + y_3}{3}$$

$$x = \frac{[ ]}{3} \text{ and } y = \frac{[ ]}{3}$$

$$\text{coordinates of centroid } x = [ ] \text{ and } y = [ ]$$

**Q. 2 B) Solve the following. 2 marks for each question**

**Example for Practice**

- 1) Find the Distance between the points A(3, 7) and B(6, 3)

**Sol. :**

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d(A, B) = \sqrt{(6-3)^2 + (3-7)^2} = d(A, B) = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25}=5$$

- 2) Determine whether the points A(-1, -1) B(0, 1) and C(1, 3) collinear

$$\text{slope of AB} = \frac{1 - (-1)}{0 - (-1)} = 2 \quad \text{slope of BC} = \frac{3 - 1}{1 - 0} = 2$$

$$\text{slope of AB} = \text{slope of BC}$$

Hence Point A and point B are collinear.

**Example for Practice**

- 1) Determine whether the points P(2, 5) Q(3, 3) and R(5, 1) collinear.

- 2) Find the Distance between the points L(-3, -2) and M(5, -8)

**Q. 3) Complete the following activity**

- 1) Find the ratio in which the line joining the points A(3, 8) and B(-9, 3) is divided by the Y-axis

Let C be a point on Y axis which divides seg AB in the Ratio m : n

C lies on Y axis C(0, y)

$$x_1 = [3] \quad y_1 = [8] \quad \text{and} \quad x_2 = [-9], \quad y_2 = [3]$$

$$\text{By section formula : } x = \frac{mx_2 + nx_1}{m + n}$$

$$0 = \frac{[-9]m + [3]n}{m + n}$$

$$\frac{m}{n} = \left[ \frac{9}{3} \right] \text{ therefore ratio } 3 : 1$$

- 2) Given that A(4, -3) B(8, 5) to find the coordinate of point that divides segAB in the ratio 3 : 1

Let the co-ordinates of point P(x, y) which divides segAB in the ratio 3 : 1

by section formula  $x = \frac{mx_2 + nx_1}{m + n}$  and  $y = \frac{my_2 + ny_1}{m + n}$

$$x = \frac{[8] \times 3 + [4] \times 1}{3 + 1} \quad \text{and} \quad y = \frac{[5] \times 3 + [-3] \times 1}{3 + 1}$$

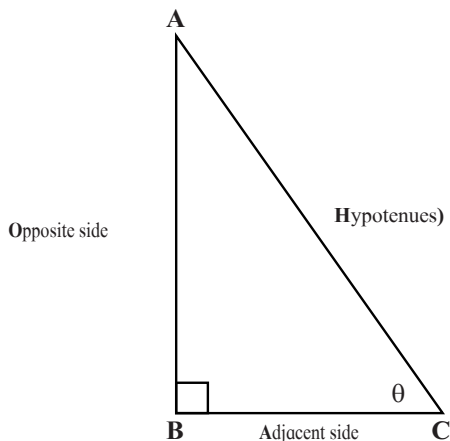
$$x = [ \quad ] \quad \text{and} \quad y = [ \quad ]$$

Therefore co-ordinates of point P(..... , .....



## 6. Trigonometry

1)



$$1) \sin \theta = \frac{O}{H} = \frac{AB}{AC}$$

$$2) \cos \theta = \frac{A}{H} = \frac{BC}{AC}$$

$$3) \tan \theta = \frac{O}{A} = \frac{AB}{BC}$$

$$4) \operatorname{cosec} \theta = \frac{H}{O} = \frac{AC}{AB}$$

$$5) \sec \theta = \frac{H}{A} = \frac{AC}{BC}$$

$$6) \cot \theta = \frac{A}{O} = \frac{BC}{AB}$$

2) The relation between the trigonometric ratios :

$$1) \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$2) \cos \theta = \frac{1}{\sec \theta}$$

$$3) \tan \theta = \frac{1}{\cot \theta}$$

$$4) \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$5) \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$6) \sec \theta = \frac{1}{\cos \theta}$$

$$7) \cot \theta = \frac{1}{\tan \theta}$$

$$8) \cot \theta = \frac{\cos \theta}{\sin \theta}$$

How to remember

Reciprocal of

$$\begin{array}{c} \downarrow \\ \sin \theta \longleftrightarrow \operatorname{cosec} \theta \\ \cos \theta \longleftrightarrow \sec \theta \\ \tan \theta \longleftrightarrow \cot \theta \end{array}$$

3) Trigonometric ratios of complementary angles :

$$1) \sin \theta = \cos (90 - \theta)$$

$$2) \tan \theta = \cot (90 - \theta)$$

$$3) \sec = \operatorname{cosec} (90 - \theta)$$

$$4) \cos \theta = \sin (90 - \theta)$$

$$5) \cot \theta = \tan (90 - \theta)$$

$$6) \operatorname{cosec} \theta = \sec (90 - \theta)$$

How to remember

$$\begin{array}{c} \downarrow \\ (90 - \theta) \\ \sin \theta \longleftrightarrow \cos \theta \\ \operatorname{cosec} \theta \longleftrightarrow \sec \theta \\ \tan \theta \longleftrightarrow \cot \theta \end{array}$$

4) Trigonometric identities :

1)  $\sin^2 \theta + \cos^2 \theta = 1$

2)  $\tan^2 \theta + 1 = \sec^2 \theta$

3)  $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$

5)

- The table of the values of trigonometric ratios of angles  $0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$

Trigonometric ratio	Angle ( $\theta$ )				
	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta$ $= \frac{1}{\sin \theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$ $= \frac{1}{\cos \theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$ $= \frac{1}{\tan \theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Q. 1 A) Choose the correct alternative from those given below each question :

(1 Mark for each MCQ)

1)  $\sin \theta \cdot \operatorname{cosec} \theta = ?$

A) 1

B) 0

C)  $\frac{1}{2}$

D)  $\sqrt{2}$

2)  $\sec 60^\circ = ?$

A)  $\frac{1}{2}$

B) 2

C)  $\frac{2}{\sqrt{3}}$

D)  $\sqrt{2}$

3)  $1 + \tan^2 \theta = ?$

A)  $\tan^2 \theta$

B)  $\sec^2 \theta$

C)  $\operatorname{cosec}^2 \theta$

D)  $\cos^2 \theta$

4)  $\sin^2 \theta + \sin^2 (90 - \theta) = ?$

A) 0

B) 1

C) 2

D)  $\sqrt{2}$

5)  $\tan \theta = \sqrt{3}$  तब  $\theta = ?$

A)  $30^\circ$

B)  $45^\circ$

C)  $60^\circ$

D)  $90^\circ$

6)  $\cot (90-\theta) = ?$

A)  $\sin \theta$

B)  $\cos \theta$

C)  $\cot \theta$

D)  $\tan \theta$

7) Value of  $2 \tan 45^\circ - 2 \sin 30^\circ$  is .....

A) 0

B) 1

C) 2

D)  $\sqrt{2}$

8) If  $\sin \theta = \frac{\sqrt{3}}{2}$  then  $\cos \theta = ?$

A)  $\frac{1}{2}$

B)  $\frac{4}{5}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{5}{3}$

**Q. 1 B) Solve the following questions :**

**(1 Mark for each question)**

1)  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = ?$

**Solution :**  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$  ..... (given)

$$= \frac{1 - 1^2}{1 + 1^2}$$

$$= \frac{0}{2} = 0$$

2) Prove that  $\operatorname{cosec} \theta \cdot \sqrt{1 - \cos^2 \theta} = 1$

**Solution :**  $\operatorname{cosec} \theta \cdot \sqrt{1 - \cos^2 \theta}$  (given)

$$= \operatorname{cosec} \theta \cdot \sqrt{\sin^2 \theta} \dots\dots (\because 1 - \cos^2 \theta = \sin^2 \theta)$$

$$= \operatorname{cosec} \theta \cdot \sin \theta$$

$$= 1$$

3) If  $\tan \theta = 1$  then  $\sin \theta \cdot \cos \theta = ?$

**Solution :**  $\tan \theta = 1$  ..... (given)

$$\therefore \tan \theta = \tan 45^\circ$$

$$\therefore \theta = 45^\circ$$

$$\text{Now consider, } \sin \theta \cdot \cos \theta = \sin 45^\circ \cdot \cos 45^\circ = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

4) If  $2 \sin \theta = 3 \cos \theta$  then  $\tan \theta = ?$

5) If  $\cot (90 - A) = 1$  then  $\angle A = ?$

6)  $\frac{\sin 75^\circ}{\cos 15^\circ} = ?$

**Q.2 A) Complete the following activities. (Write complete answer)**

**(2 Marks for each question)**

1)  $\cos^2 \theta \cdot (1 + \tan^2 \theta) = 1$ , for proof of this complete the activity given below.

**Activity :** L. H. S =   
=  $\cos^2 \theta \times$   .... ( $1 + \tan^2 \theta =$  )  
=  $(\cos \theta \times$  )<sup>2</sup>  
=  $1^2$   
=  $1 = \text{R. H. S}$

**Solution :** L. H.S =   
=  $\cos^2 \theta \times$   .... ( $1 + \tan^2 \theta =$  )  
=  $(\cos \theta \times$  )<sup>2</sup>  
=  $1^2$   
=  $1 = \text{R. H. S}$

2)  $\frac{5}{\sin^2 \theta} - 5 \cot^2 \theta$  For proof of value of this complete the activity given below.

**Activity :**  $\frac{5}{\sin^2 \theta} - 5 \cot^2 \theta$   
=   $\left( \frac{1}{\sin^2 \theta} - \cot^2 \theta \right)$   
=  $5 \left( \right.$    $\left. - \cot^2 \theta \right)$  .... ( $\frac{1}{\sin^2 \theta} =$  )  
=  $5 (1)$   
=

**Solution :**  $\frac{5}{\sin^2 \theta} - 5 \cot^2 \theta$  .... (given)  
=   $\left( \frac{1}{\sin^2 \theta} - \cot^2 \theta \right)$   
=  $5 \left( \right.$    $\left. - \cot^2 \theta \right)$  .... ( $\because \frac{1}{\sin^2 \theta} =$  )  
=  $5 (1) =$



3) If  $\sec \theta + \tan \theta = \sqrt{3}$  then  $\sec \theta - \tan \theta$ . To find value of this complete the activity given below.

**Activity :**  =  $1 + \tan^2 \theta$  ..... (Fundamental tri. identity)

$$\text{input} - \tan^2 \theta = 1$$

$$(\sec \theta + \tan \theta) \cdot (\sec \theta - \tan \theta) = \text{input}$$

$$\sqrt{3} \cdot (\sec \theta - \tan \theta) = 1$$

$$(\sec \theta - \tan \theta) = \text{input}$$

**Solution :**  =  $1 + \tan^2 \theta$  ..... (Fundamental tri. identity)

$$\therefore \text{input} - \tan^2 \theta = 1$$

$$(\sec \theta + \tan \theta) \cdot (\sec \theta - \tan \theta) = \text{input}$$

$$\sqrt{3} (\sec \theta - \tan \theta) = 1 \text{ ..... } (\sec \theta + \tan \theta = \sqrt{3} \text{ given})$$

$$(\sec \theta - \tan \theta) = \frac{1}{\sqrt{3}}$$

4) If  $\tan \theta = \frac{9}{10}$  then  $\sec \theta$ . To find value of this complete the activity given below.

**Activity :**  $\sec^2 \theta = 1 + \frac{\text{input}}{\text{input}}$  ..... (Fundamental tri. identity)

$$\sec^2 \theta = 1 + \frac{\text{input}}{\text{input}^2}$$

$$\sec^2 \theta = 1 + \frac{\text{input}}{\text{input}}$$

$$\sec \theta = \text{input}$$

**Q. 2 B) Solve the following questions :**

**(2 Marks for each question)**

1) If  $\cos \theta = \frac{24}{24}$  then  $\sin \theta = ?$

**Solution :**  $\cos \theta = \frac{24}{24}$  .... (given)

$$\therefore \cos \theta = 1$$

$$\therefore \cos \theta = \cos 0^\circ$$

$$\therefore \theta = 0^\circ$$

$$\text{Now, } \sin \theta = \sin 0^\circ$$

$$= 0$$

2) Prove that  $\frac{\sin^2 \theta}{\cos \theta} + \cos \theta = \sec \theta$ .

**Solution :** given L. H. S

$$= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} \dots (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= \sec \theta$$

$$= \text{R. H. S}$$

3) prove that - If  $\cos (45^\circ + x) = \sin 30^\circ$  then  $x = ?$

4) prove that - If  $\tan \theta + \cot \theta = 2$  then  $\tan^2 \theta + \cot^2 \theta = ?$

5) Prove that  $\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \times \operatorname{cosec}^2 \theta$ .

6) Prove that  $\cot^2 \theta \times \sec^2 \theta = \cot^2 \theta + 1$ .

7) Prove that - If  $\sin 3A = \cos 6A$  then  $\angle A = ?$

8) Prove that  $\frac{\tan A}{\cot A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$ .

**Q. 3 A) Complete the following activities.**

**(3 Marks for each question)**

1)  $\sin^4 A - \cos^4 A = 1 - 2 \cos^2 A$ , for proof of value of this complete the activity given below.

$$\begin{aligned}
 \text{Activity : L. H. S} &= \boxed{\phantom{000000}} \\
 &= (\sin^2 - \cos^2 A) (\boxed{\phantom{000000}}) \\
 &= 1 \boxed{\phantom{000000}} \dots (\sin^2 A + \boxed{\phantom{000000}} = 1) \\
 &= \boxed{\phantom{000000}} - \cos^2 A \dots (\sin^2 A = 1 - \cos^2 A) \\
 &= \boxed{\phantom{000000}} \\
 &= \text{R. H. S}
 \end{aligned}$$

$$\begin{aligned}
 \text{Solution : L. H. S} &= (\boxed{\sin^4 A - \cos^4 A}) \\
 &= (\sin^2 + \cos^2 A) (\boxed{\sin^2 A - \cos^2 A}) \\
 &= 1 (\boxed{\sin^2 A - \cos^2 A}) \dots (\sin^2 A + \boxed{\cos^2 A} = 1) \\
 &= \boxed{1 - \cos^2 A} - \cos^2 A \dots (\sin^2 A = 1 - \cos^2 A) \\
 &= \boxed{1 - 2\cos^2 A} \\
 &= \text{R. H. S}
 \end{aligned}$$

2)  $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \times \sin^2 \theta$  to find value of this complete the activity given below.

$$\begin{aligned}
 \text{Activity : L. H. S} &= \boxed{\phantom{000000}} \\
 &= \boxed{\phantom{000000}} \left( 1 - \frac{\sin^2 \theta}{\tan^2 \theta} \right) \\
 &= \tan^2 \theta \left( 1 - \frac{\boxed{\phantom{000000}}}{\frac{\sin^2 \theta}{\cos^2 \theta}} \right) \\
 &= \tan^2 \theta \left( 1 - \frac{\sin^2 \theta}{1} \times \frac{\cos^2 \theta}{\boxed{\phantom{000000}}} \right)
 \end{aligned}$$

$$\begin{aligned}
&= \tan^2 \theta \left( 1 - \boxed{\phantom{000000}} \right) \\
&= \tan^2 \theta \times \boxed{\phantom{000000}} \left( 1 - \cos^2 \theta = \sin^2 \theta \right) \\
&= \text{R. H. S}
\end{aligned}$$

**Solution :** L. H. S =  $\boxed{\tan^2 \theta - \sin^2 \theta}$

$$\begin{aligned}
&= \boxed{\tan^2 \theta} \left( 1 - \frac{\sin^2 \theta}{\tan^2 \theta} \right) \\
&= \tan^2 \theta \left( 1 - \frac{\sin^2 \theta}{\cos^2 \theta} \right) \\
&= \tan^2 \theta \left( 1 - \frac{\sin^2 \theta}{1} \times \frac{\cos^2 \theta}{\sin^2 \theta} \right) \\
&= \tan^2 \theta \left( 1 - \boxed{\cos^2 \theta} \right) \\
&= \tan^2 \theta \times \boxed{\sin^2 \theta} \dots\dots \left( - 1 \cos^2 \theta = \sin^2 \theta \right) \\
&= \text{R. H. S}
\end{aligned}$$

3) If  $\tan \theta = \frac{7}{24}$  than  $\cos \theta$  To find value of  $\cos \theta$  complete the activity given below.

**Activity :**  $\sec^2 \theta = 1 + \boxed{\phantom{000000}} \dots\dots$  (Fundamental tri. identity)

$$\sec^2 \theta = 1 + \boxed{\phantom{000000}}^2$$

$$\sec^2 \theta = 1 + \frac{\boxed{\phantom{000000}}}{576}$$

$$\sec^2 \theta = \frac{\boxed{\phantom{000000}}}{576}$$

$$\sec \theta = \boxed{\phantom{000000}}$$

$$\cos \theta = \boxed{\phantom{000000}} \dots\dots \left( \cos \theta = \frac{1}{\sec \theta} \right)$$

4)  $\cot \theta + \tan \theta = \operatorname{cosec} \theta \times \sec \theta$  For proof of this complete the activity given below.

$$\begin{aligned}
 \text{Activity : L. H. S} &= \boxed{\phantom{000000}} \\
 &= \frac{\boxed{\phantom{000000}}}{\sin \theta} = \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\boxed{\phantom{000000}}} \\
 &= \frac{1}{\sin \theta \cdot \cos \theta} \dots \dots \left( \cos^2 \theta + \sin^2 \theta = \boxed{\phantom{000000}} \right) \\
 &= \frac{1}{\sec \theta} \times \frac{1}{\boxed{\phantom{000000}}} \\
 &= \boxed{\phantom{000000}} \\
 &= \text{R. H. S}
 \end{aligned}$$

**Q. 3 B) Solve the following questions :**

**(3 Marks for each question)**

1) Prove that  $\frac{\tan(90 - \theta) + \cot(90 - \theta)}{\operatorname{cosec} \theta} = \sec \theta$

**Solution :** Consider L. H. S

$$\begin{aligned}
 &= \frac{\tan(90 - \theta) + \cot(90 - \theta)}{\operatorname{cosec} \theta} \dots \dots \text{(given)} \\
 &= \frac{\cot \theta + \tan \theta}{\operatorname{cosec} \theta} \left[ \because \tan \theta = \cot(90 - \theta) \text{ and } \cot \theta = \tan(90 - \theta) \right] \\
 &= \frac{\frac{\cos^2 \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\sin \theta}} \left( \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right) \\
 &= \frac{\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta}}{\frac{1}{\sin \theta}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{1}{\sin \theta \cdot \cos \theta}}{\frac{1}{\sin \theta}} \left( \because \cos^2 \theta + \sin^2 \theta = 1 \right) \\
&= \frac{1}{\cos \theta} \\
&= \sec \theta \left( \because \frac{1}{\cos \theta} = \sec \theta \right) \\
&= \text{R. H. S}
\end{aligned}$$

2) Prove that -  $\frac{\sec A}{\tan A + \cot A} = \sin A$

3) Prove that -  $\frac{\sin \theta + \operatorname{cosec} \theta}{\sin \theta} = 2 + \cot^2 \theta$

4) Prove that  $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$

5) Prove that  $\sec^2 \theta - \cos^2 \theta = \tan^2 \theta + \sin^2 \theta$

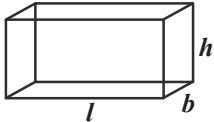
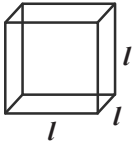

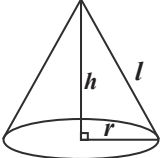
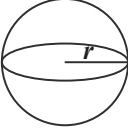



6) Prove that - In  $\Delta ABC$ ,  $\cos C = \frac{12}{13}$  and  $BC = 24$  then  $AC = ?$

7) Prove that - If,  $\sin A = \frac{3}{5}$  then show that  $4 \tan A + 3 \tan A = 6 \cos A$ .

## 7. Mensuration

Previous Knowledge :

Solid Figure	Surface Area			
	Vertical	Curved	Total Surface	Volume
<b>Cuboid</b> 	$2h(l + b)$	-	$2(lb + bh + hl)$	$(l \times b \times h)$
<b>Cube</b> 	$4l^2$	-	$6l^2$	$l^3$
<b>Cylinder</b> 	-	$2\pi r^2$	$2\pi r(r + h)$	$\pi r^2 h$
<b>Cone</b> 	-	$\pi r l$ where, $l = \sqrt{h^2 + r^2}$	$\pi r(l + r)$	$\frac{1}{3} \pi r^2 h$
<b>Sphere</b> 	-	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3} \pi r^3$
<b>Hemisphere</b> 	-	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3} \pi r^3$

• Area of sector =  $\frac{\theta}{360} \times \pi r^2$

• Length of arc =  $l = \frac{\theta}{360} \times 2\pi r$

• Area of Sector =  $\frac{1}{2} \times l \times r$

where,  $l$  = Length of arc

Solve :

**Q.1. A ) Four alternative answers are given for every sub question. Select the correct alternative and write the alphabet of that answer.**

- 1) Find the volume of a cube of side 3 cm.  
A)  $27 \text{ cm}^3$       B)  $9 \text{ cm}^3$       C)  $81 \text{ cm}^3$       D)  $3 \text{ cm}^3$
- 2) Find the perimeter of a sector of a circle if its measure is  $90^\circ$  and radius is 7 cm.  
A) 44cm      B) 25 cm      C) 36 cm      D) 56 cm
- 3) Find the curved surface area of a cone of radius 7 cm and height 24 cm.  
A)  $440 \text{ cm}^2$       B)  $330 \text{ cm}^2$       C)  $110 \text{ cm}^2$       D)  $550 \text{ cm}^2$
- 4) A circle having radius 4 cm, then the length of its largest chord is.....  
A) 2 cm      B) 4 cm      C) 12 cm      D) 8 cm
- 5) What is the ratio of the volumes of a cylinder and cone having equal radius and equal height.  
A) 1 : 3      B) 3 : 1      C) 1 : 2      D) 2 : 1
- 6) If the radius of the base of cone is 5cm and height is 12 cm ,then its slant height is --  
A) 23 cm      B) 31 cm      C) 13 cm      D) 12 cm

**Q. 1 B) Solve the following questions.**

- 1) If the measure of an arc of a circle is  $160^\circ$  and its length is 44 cm, find the circumference of the circle.

**Ans. :** Measure of the arc =  $\theta = 160^\circ$

$$\text{Length of arc} = l = 44 \text{ cm}$$

$$\text{Length of arc} = l = \frac{\theta}{360} \times 2\pi r$$

$$44 = \frac{160}{360} \times \text{circumference of the circle}$$

$$44 = \frac{4}{9} \times \text{circumference of the circle}$$

$$\frac{44 \times 9}{4} = \text{circumference of the circle}$$

$$99 = \text{circumference of the circle}$$

Thus, circumference of the circle = 99 सेमी

2) If the radius of cylinder is 7 cm and height is 2 cm then find its volume.

3) The volume of a cube is  $1000 \text{ cm}^3$ . Find the side of a cube.

**Q. 2 A) Complete the following activities and rewrite it.**

1) The radius of the sphere is 7 cm. Complete the following activity to find its surface area.

$$\begin{aligned}\text{Activity : Surface area of the sphere} &= 4 \pi r^2 \\ &= 4 \times \frac{22}{7} \times \boxed{\phantom{00}}^2 \\ &= 4 \times \frac{22}{7} \times \boxed{\phantom{00}} \\ &= \boxed{\phantom{00}} \times 7 \\ &= \boxed{\phantom{00}} \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Activity : Surface arc of the sphere} &= 4 \pi r^2 \\ &= 4 \times \frac{22}{7} \times \boxed{7}^2 \\ &= 4 \times \frac{22}{7} \times \boxed{49} \\ &= \boxed{88} \times 7 \\ &= \boxed{616} \text{ cm}^2\end{aligned}$$

- 2) Complete the following activity to find the volume of a sphere if its diameter is 6 cm.

**Activity :** The radius of the sphere =  $r = \frac{6}{2} = \boxed{\phantom{00}}$  cm

Volume of a Sphere =  $\boxed{\phantom{00}}$  ..... (Formula)

$= \frac{4}{3} \times 3.14 \times \boxed{\phantom{00}}$

$= \boxed{\phantom{00}} \text{ cm}^3$

- 3) A tank of cylinder in shape has radius 2.8 m and its height 3.5 m. Complete the activity to find how many litres of water the tank will contain.

**Activity :** Capacity of water tank = Volume of cylindrical tank

$= \boxed{\phantom{000000}}$  ..... (Formula)

$= \frac{22}{7} \times 2.8 \times 2.8 \times 3.5$

$= \boxed{\phantom{000000}} \text{ m}^3$

$= \boxed{\phantom{000000}} \times 1000 \text{ litre}$

$= \boxed{\phantom{000000}} \text{ litre}$

**Q. 2 B) Solve the following sub-questions.**

- 1) The length of an arc of a circle is 10 cm and radius is 5 cm. Find the area of the sector.

length of an arc  $l = 10 \text{ cm}$

Radius of the sector  $r = 5 \text{ cm}$

Area of sector  $= \frac{1}{2} \times l \times r$

$= \frac{1}{2} \times 10 \times 5$

$= 25 \text{ cm}^2$

Thus, the area of the sector is  $= 25 \text{ cm}^2$

2) The radius of cone is 7 cm. Find its volume? ( $\pi \frac{22}{7}$ )

3) The dimensions of a cuboid in cm are  $30 \times 18 \times 10$ . Find its volume.

4) The radius of the base of a right circular cylinder is 3 cm and height is 7 cm, find the curved surface area. ( $\pi \frac{22}{7}$ )

**Q. 3 A) Complete the following activities and rewrite it.**

- 1) Complete the following activity to find the radius of hemisphere if its volume is 2094 cm<sup>3</sup>.

**Activity :** Volume of a hemisphere =  ..... (formula)

$$\text{Volume} = \frac{2}{3} \times \pi r^3$$

$$\therefore r^3 = \frac{\text{Volume} \times 3}{2 \times \pi}$$

$$\therefore r^3 = 1000$$

$$\therefore r = 10 \text{ cm}$$

**Activity :** Volume of a hemisphere =  $\frac{2}{3} \times \pi r^3$  ..... (formula)

$$2094 = \frac{2}{3} \times 3.14 \times r^3$$

$$r^3 = \frac{2094 \times 3}{2 \times 3.14}$$

$$r^3 = 1000$$

$$r = 10 \text{ cm}$$

- 2) A metal parallelepiped of measures 16cm × 11cm × 10 cm were melted to make coins. How many coins were made if the thickness and diameter of each coin was 2 mm and 2 cm respectively ? Complete the following activity.

**Activity :** The length of a metal parallelopiped  $l = 16 \text{ cm}$ ,

It's breadth  $b = 11$

It's height  $h = 10 \text{ cm}$

The diameter of coin = 2 mm.

Its radius =  $r = 1 \text{ cm}$

Its thickness =  $h_1 = 0.2 \text{ cm}$

$$\text{Number of coins made} = \frac{\text{Volume of metal parallelopiped}}{\text{volume of each coin}}$$

$$= \frac{\boxed{\phantom{16 \times 11 \times 10}}}{\boxed{\phantom{22 \times 2}}} \dots\dots\dots \text{(Formula)}$$

$$= \frac{16 \times 11 \times 10}{\phantom{22 \times 2}}$$

$$= \frac{\frac{22}{7} \times 1 \times 1 \times \boxed{\phantom{7 \times 10}}}{\phantom{22 \times 2}}$$

$$= \frac{16 \times 11 \times 10 \times 7 \times 10}{22 \times 2}$$

$$\text{Total no. of coins} = \boxed{\phantom{16 \times 11 \times 10 \times 7 \times 10 / (22 \times 2)}}$$

**Q. 3 B) Solve the following subquestions.**

1) Find the volume and surface area of a sphere of radius 2.1 cm ( $\pi \frac{22}{7}$ )

2) The volume of cube is  $1000 \text{ cm}^3$ . Find its total surface area.

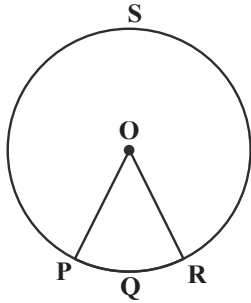


3) In the figure, radius of the circle is 7 cm and  $m(\text{arc PQR}) = 60^\circ$ . Find

i) Area of the circle

ii)  $A(O - PQR)$

iii)  $A(O - PSR)$



## Std. X - Subject : Mathematics

### Part - I and II

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